We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

31

Practical Monte Carlo Based Reliability Analysis and Design Methods for Geotechnical Problems

Jianye Ching
National Taiwan University, Taiwan, Republic of China

1. Introduction

Reliability analysis is an important tool for quantifying uncertainties in analysis and design of engineering systems. In the past decades, the so-called first-order reliability method (FORM) (Ang & Tang, 1984) was the main stream method for reliability analysis. This method transforms a reliability analysis problem into an approximate optimization problem so that the required computation is minimized. Nonetheless, such transformation comes with some premises and tradeoffs: (a) to make the optimization problem tractable, the number of random variables of the target problem cannot be too many; (b) the problem at hand is better to be lightly nonlinear to avoid large bias in the estimated reliability; and (c) the engineers must have basic skills for solving nonlinear optimization problems.

The first two premises may be questionable for realistic geotechnical problems because there are typically numerous random variables in realistic geotechnical engineering analyses and designs. Although techniques are developed to reduce the number of random variables (e.g., Ghanem & Spanos, 1991), their generality and accuracy are not yet proved. Therefore, for realistic geotechnical engineering analyses and designs, FORM may not be the best solution. More seriously, average engineers may not have the knowledge and skills for nonlinear optimization. It is not trivial for them to implement FORM, even for the simplest geotechnical design examples.

Given the rapid growth of nowadays personal computers (PCs), massive computations are now more possible than ever. In particular, Monte Carlo simulations (MCS) can nowadays be implemented for the purpose of reliability analyses even with PCs. MCS is general for the number of random variables and the problem complexity; hence the limitation of FORM can be easily overcome. Moreover, the basic idea of MCS is very simple and intuitive. Finally, geotechnical models can be treated as black boxes when implementing MCS. All these features make MCS attractive for practicality. The only criticism for MCS is that it is inefficient for problems with very small failure probabilities (or with very high reliabilities). However, this limitation has been gradually removed by the recent advancements in the Monte Carlo based reliability methods.

The goal of this chapter is to demonstrate the uses of some Monte Carlo based reliability methods and reliability-based design methods. In particular, a realistic geotechnical design example is developed for the purpose of demonstration: the implementation of all methods

will be presented based on the same example. First of all, this chapter will review practical Monte Carlo based reliability analysis methods, including

a. Direct Monte Carlo simulation
b. Importance sampling
c. Subset simulation

The traditional FORM will be also briefly reviewed for completeness.

Second, this chapter will review state-of-the-art developments in the Monte Carlo based reliability-based design methods. This subject is the inverse problem of reliability analyses: the purpose of reliability analyses is to obtain the reliability given the design dimension of an engineering system, but the purpose of reliability-based design is to obtain the design dimension given the target reliability. The author himself (Ching & Phoon, 2010) has developed a series of Monte Carlo based methods in this line. The review will be limited to the following design methods:

a. Monte Carlo based safety factor design
b. Monte Carlo based load-resistance factor design
c. Monte Carlo based multiple resistance factor design
d. Monte Carlo based partial factor design

As opposed to the FORM-based reliability-based design methods, these Monte Carlo based methods are, again, not limited by the number of random variables and problem complexity and not requiring the acknowledge of optimization skills.

2. Design example for demonstration

Throughout this chapter, an example of geotechnical designs will be given to demonstrate the reviewed reliability analysis and reliability-based design methodologies. Consider a drilled shaft of 74.7 m long that is to be built at a site with ground profile shown in Table 1,

Type	Depth (m)	Middle depth d (m)	$\sigma'_{v,m}$ (kN/m^2)	Thickness t (m)	$s_{u,m}$ (kN/m^2)	$q_{u,m}$ (kN/m^2)
Clay	0.0 – 42.1			42.1 = t_c	70	
Sand	42.1 – 49.6	45.85 = d_s	350 = $\sigma'_{vs,m}$	7.5 = t_s		
Gravel	49.6 – 69.7	59.65 = d_g	480 = $\sigma'_{vg,m}$	20.1 = t_g		
Sandstone	69.7 – 74.7			5.0 = t_r		900

Table 1. Ground profile for the example design site

where there are four strata, including clay 'c', sand 's', gravel 'g', and rock 'r' layers; d_x and t_x respectively denote the middle depth and thickness of each layer (the subscript 'x' may be either 'c', 's', 'g', or 'r', depending on the associated stratum type); $\sigma'_{vs,m}$, $\sigma'_{vg,m}$, $s_{u,m}$, and $q_{u,m}$ are respectively the measured in-situ effective stress in sand layer, in-situ effective stress in gravel layer, undrained shear strength of clay layer, and uniaxial compression strength of rock layer. The measurement is subjected to measurement errors:

$$\begin{aligned} &\ln\left(\sigma'_{vs,m}\right) = \ln\left(\sigma'_{vs}\right) + e_{\sigma'_{vs}} \qquad \ln\left(\sigma'_{vg,m}\right) = \ln\left(\sigma'_{vg}\right) + e_{\sigma'_{vg}} \\ &\ln\left(s_{u,m}\right) = \ln\left(s_u\right) + e_{s_u} \qquad \ln\left(q_{u,m}\right) = \ln\left(q_u\right) + e_{q_u} \end{aligned} \tag{1}$$

where σ'_{vs} , σ'_{vg} , s_u , q_u are the corresponding actual values, and $e_{\sigma'vs}$, $e_{\sigma'vg}$, e_{su} , e_{qu} quantify measurement errors. These measurement errors are modeled as zero-mean normal random variables with standard deviations chosen to be 0.1, 0.1, 0.2, and 0.5, respectively.
The axial compression capacity (Q) of the drilled shaft is provided by the side resistance (S) and tip resistance (T), and it can be computed using the equation given below:

$$Q = S + T \tag{2}$$

Although the shaft tip may contribute to the overall compression resistance, the majority of the compression capacity is provided by the side resistance. Its contribution to the overall capacity often is ignorable compared to the side resistance. The side resistance is provided by the shaft adhesion for cohesive soils and rocks and shaft frictional resistance for cohesionless soils:

$$S = S_c + S_s + S_g + S_r \tag{3}$$

where S_c, S_s, S_g, and S_r are side resistances for the clay, sand, gravel, and rock layers, respectively. The side resistance in a given layer, denoted by S_x, can be computed as

$$S_x = \pi B f_{sx} t_x \tag{4}$$

where B is the diameter of the shaft; f_{sx} is the unit side resistance provided by layer 'x'. The unit side resistance f_s is correlated to geotechnical parameters such as s_u, σ'_v, and q_u. Useful empirical correlation equations are listed in Table 2, where ε's quantify the transformation uncertainties. These ε's are modeled as zero-mean normal random variables with standard deviations listed in the table. It is then clear that

$$\begin{aligned}
S_c &= \pi B e^{2.7+0.3\ln s_u+\varepsilon_{S_c}} t_c = \pi B e^{2.7+0.3\left[\ln s_{u,m}-e_{s_u}\right]+\varepsilon_{S_c}} t_c \\
S_s &= \pi B e^{1.0802-0.6588\ln(d_s)+\ln(\sigma'_{vs})+\varepsilon_{S_s}} t_s = \pi B e^{1.0802-0.6588\ln(d_s)+\left[\ln(\sigma'_{vs,m})-e_{\sigma'_{vs}}\right]+\varepsilon_{S_s}} t_s \\
S_g &= \pi B e^{2.1792-0.7528\ln(d_g)+\ln(\sigma'_{vg})+\varepsilon_{S_g}} t_g = \pi B e^{2.1792-0.7528\ln(d_g)+\left[\ln(\sigma'_{vg,m})-e_{\sigma'_{vg}}\right]+\varepsilon_{S_g}} t_g \\
S_r &= \pi B e^{3.0253+0.414\ln q_u+\varepsilon_{S_r}} t_r = \pi B e^{3.0253+0.414\left[\ln q_{u,m}-e_{q_u}\right]+\varepsilon_{S_r}} t_r
\end{aligned} \tag{5}$$

Reliability analyses and reliability-based designs will be demonstrated on this design example. The drilled shaft is subjected to an axial deal load L_D and axial live load L_L. They are modeled as lognormal random variables with mean values $\{\mu_{LD}, \mu_{LL}\}$ and coefficients of variation (c.o.v.) $\{\delta_{LD} = 0.1, \delta_{LL} = 0.25\}$. The herein goal is to demonstrate (a) the calculation of the reliability of a drilled shaft with given dimension (i.e., diameter B = 1.2 m and length L = 74.7 m) and to demonstrate (b) the determination of the required dimension B and L to achieve a prescribed target reliability. The item (a) is the goal for reliability analysis, while item (b) is for reliability-based design.
The collection of random variables is denoted by $X \in R^p$, where p is the dimension of X. For this example, X includes the measurement errors $\{e_{\sigma'vs}, e_{\sigma'vg}, e_{su}, e_{qu}\}$, transformation uncertainties $\{e_{Sc}, e_{Ss}, e_{Sg}, e_{Sr}\}$ and loads $\{L_D, L_L\}$. The collection of design parameters is denoted by $\theta \in R^q$, where q is the dimension of θ. For this example, θ includes the diameter B and shaft length L. Let F denotes the failure event: $F=\{SR(X,\theta)<1\}$, where $SR(X,\theta)$ is called the

safety ratio, the random version of the classical safety factor. In general, a safety ratio less than 1 does not necessarily imply the complete collapse of the system but does imply unsatisfactory performance of the system in the sense of violating some limit states, e.g. serviceability, repairable, or ultimate limit states. Throughout the chapter, it can be assumed without loss of generality that SR(X,θ) is positive and that the probability density function (PDF) of the random vector X conditioned on θ (i.e., θ takes specific numerical values), denoted by p(x | θ), is known.

	Correlation Model for Unit Side Resistance f_s (kN/m²)	Standard Deviation of Transformation Uncertainty ε
Clay	$f_s = \exp\left(2.7 + 0.3\ln(s_u) + \varepsilon_{S_c}\right)$	0.3216
Sand	$f_s = \exp\left(1.0802 - 0.6588\ln(d) + \ln(\sigma_v^{'}) + \varepsilon_{S_s}\right)$	0.5414
Gravel	$f_s = \exp\left(2.1792 - 0.7528\ln(d) + \ln(\sigma_v^{'}) + \varepsilon_{S_g}\right)$	0.6689
Rock	$f_s = \exp\left(3.0253 + 0.414\ln(q_u) + \varepsilon_{S_r}\right)$	0.7160

Table 2. Correlation models for evaluating unit side resistance and the associated uncertainty for various strata

For this particular example, the safety ratio SR(X,θ) can be defined as:

$$SR = \frac{S_c + S_s + S_g + S_r}{L_D + L_L} \tag{6}$$

As will be clear later, it is convenient to transform the entire problem into the standard Gaussian space, i.e.,

$$\begin{aligned} SR(Z,\theta) &= \frac{\pi B\left[\begin{array}{l} e^{2.7+0.3\left[\ln s_{u,m}-0.2z_{s_u}\right]+0.3216z_{S_c}}t_c + e^{1.0802-0.6588\ln(d_s)+\left[\ln(\sigma_{vs,m}^{'})-0.1z_{\sigma'_{vs}}\right]+0.5414z_{S_s}}t_s \\ +e^{2.1792-0.7528\ln(d_g)+\left[\ln(\sigma_{vg,m}^{'})-0.1z_{\sigma'_{vg}}\right]+0.6689z_{S_g}}t_g + e^{3.0253+0.414\left[\ln q_{u,m}-0.5z_{q_u}\right]+0.7160z_{S_r}}t_r \end{array}\right]}{e^{\ln\left(\mu_{LD}/\sqrt{1+0.1^2}\right)+\sqrt{\ln\left(1+0.1^2\right)}\cdot z_{LD}} + e^{\ln\left(\mu_{LL}/\sqrt{1+0.25^2}\right)+\sqrt{\ln\left(1+0.25^2\right)}\cdot z_{LL}}} \\ &= \frac{\pi B\left[\begin{array}{l} e^{2.7+0.3\left[\ln(70)-0.2z_{s_u}\right]+0.3216z_{S_c}}\cdot 42.1 + e^{1.0802-0.6588\ln(45.85)+\left[\ln(350)-0.1z_{\sigma'_{vs}}\right]+0.5414z_{S_s}}\cdot 7.5 \\ +e^{2.1792-0.7528\ln(59.65)+\left[\ln(480)-0.1z_{\sigma'_{vg}}\right]+0.6689z_{S_g}}\cdot 20.1 + e^{3.0253+0.414\left[\ln(900)-0.5z_{q_u}\right]+0.7160z_{S_r}}\cdot 5.0 \end{array}\right]}{e^{\ln\left(\mu_{LD}/\sqrt{1+0.1^2}\right)+\sqrt{\ln\left(1+0.1^2\right)}\cdot z_{LD}} + e^{\ln\left(\mu_{LL}/\sqrt{1+0.25^2}\right)+\sqrt{\ln\left(1+0.25^2\right)}\cdot z_{LL}}} \end{aligned} \tag{7}$$

where Z = {$z_{\sigma'vs}$, $z_{\sigma'vg}$, z_{su}, z_{qu}, z_{Sc}, z_{Ss}, z_{Sg}, z_{Sr}, z_{LD}, z_{LL}} are jointly standard Gaussian random variables, i.e.,

$$p(z \mid \theta) = \frac{1}{2\pi^{p/2}} e^{-\frac{1}{2}z^T z} \tag{8}$$

$\{\sigma'_{vs,m}, \sigma'_{vg,m}, s_{u,m}, q_{u,m}, d_s, d_g, t_c, t_s, t_g, t_r\}$ are known numbers that can be found in Table 2, $\{\mu_{LD}, \mu_{LL}\}$ are prescribed load mean values, and $\theta = \{B, L\}$ are the design parameters.

3. Reliability analysis

Let us now consider the following drilled shaft constructed at the site: B = 1.2 m, L = 74.7 m, and also let μ_{LD} = 8000 kN and μ_{LL} = 4000 kN. Four reliability methods will be presented in this chapter to determine the reliability of this particular shaft, including (a) Direct Monte Carlo simulation (MCS) (Ang & Tang, 1984); (b) first-order reliability method (FORM) (Hasofer & Lind, 1974; Der Kiureghian, 2000; Liu and Der Kiureghian, 1991); (c) importance sampling (IS) (Melchers, 1989; Hohenbichler & Rackwitz, 1988; Der Kureghian & Dakessian, 1998; Au et al., 1999); and (d) subset simulation (Subsim) (Au & Beck, 2001). Note that only these four methods are reviewed due to their simplicity and practicality. More sophisticated methods are not the main theme of this chapter. The FORM is not a Monte Carlo based method. It is presented here because the IS method requires the FORM solution.

By definition, reliability is unity minus failure probability. As a result, central to reliability analysis is the determination of the failure probability, the probability that the failure event F occurs, denoted by $P(F|\theta)$. The failure probability can be found by evaluating the following integral:

$$P(F|\theta) = \int 1\left(SR(z,\theta) < 1\right) p(z|\theta)\, dz \tag{9}$$

where 1(.) is the indicator function: it is unity if the statement is true and is zero otherwise. When the Z dimension (p) is high, the numerical solution for this integral is typically infeasible. A possible remedy is to adopt the Monte Carlo simulation to evaluate this integral.

3.1 Direct Monte Carlo simulation

According to the Law of Large Number (Ang and Tang, 1984), the integral can be approximately evaluated as follows:

$$P(F|\theta) \approx \frac{1}{N}\sum_{i=1}^{N} 1(SR(Z^{(i)},\theta) < 1) \equiv P_F^{MCS} \tag{10}$$

where N is the total number of MCS independent samples; $Z^{(i)}$ is the i-th sample of Z, drawn from the jointly standard Gaussian distribution $p(z|\theta)$; P_F^{MCS} is the estimator for $P(F|\theta)$ based on MCS. This estimator is unbiased, i.e., the expected value of P_F^{MCS} is exactly $P(F|\theta)$, and is with c.o.v. = $\{[1- P(F|\theta)]/N/P(F|\theta)\}^{0.5}$. Note that the c.o.v. does not depend on Z dimension and does not depend on the complexity of the problem, either. This is the key advantage for MCS, especially for geotechnical problems where nonlinearity and uncertainty dimension is usually high.

The disadvantage of MCS is that it may require a large sample size when $P(F|\theta)$ is small. A rule of thumb is that it requires $N = 10/P(F|\theta)$ to achieve a reasonable accuracy, i.e., c.o.v. = 30%. The disadvantage is acceptable when the calculation of SR is fast, e.g., the example design problem for drilled shaft. For problems where a single calculation of SR is time consuming, e.g., finite element analysis, MCS may be infeasible.

For the example design problem of a drilled shaft, the following steps can be taken to estimate $P(F|\theta)$:

a. Draw N independent samples of $Z^{(i)} = \{z^{(i)}_{\sigma'vs}, z^{(i)}_{\sigma'vg}, z^{(i)}_{su}, z^{(i)}_{qu}, z^{(i)}_{Sc}, z^{(i)}_{Ss}, z^{(i)}_{Sg}, z^{(i)}_{Sr}, z^{(i)}_{LD}, z^{(i)}_{LL}\}$ from the jointly standard Gaussian distribution.

b. For each sample set, evaluate

$$SR\left(Z^{(i)},\theta\right) = \frac{\pi B\left[e^{2.7+0.3\left[\ln(70)-0.2z^{(i)}_{s_u}\right]+0.3216z^{(i)}_{S_c}}\cdot 42.1 + e^{1.0802-0.6588\ln(45.85)+\left[\ln(350)-0.1z^{(i)}_{\sigma'_{vs}}\right]+0.5414z^{(i)}_{S_s}}\cdot 7.5 + e^{2.1792-0.7528\ln(59.65)+\left[\ln(480)-0.1z^{(i)}_{\sigma'_{vg}}\right]+0.6689z^{(i)}_{S_g}}\cdot 20.1 + e^{3.0253+0.414\left[\ln(900)-0.5z^{(i)}_{q_u}\right]+0.7160z^{(i)}_{S_r}}\cdot 5.0\right]}{e^{\ln\left(8000/\sqrt{1+0.1^2}\right)+\sqrt{\ln\left(1+0.1^2\right)}\cdot z^{(i)}_{LD}} + e^{\ln\left(4000/\sqrt{1+0.25^2}\right)+\sqrt{\ln\left(1+0.25^2\right)}\cdot z^{(i)}_{LL}}} \tag{11}$$

c. Let

$$P_F^{MCS} = \frac{1}{N}\sum_{i=1}^{N} 1(SR(Z^{(i)},\theta) < 1) \tag{12}$$

By taking $N = 10^6$, P_F^{MCS} is found to be around 5.9×10^{-4}. Note that P_F^{MCS} is in general not the same as the actual $P(F|\theta)$ but is only its estimator. It will be informative to also know the c.o.v. of P_F^{MCS}. This c.o.v. can be estimated as $\{[1- P_F^{MCS}]/N/P_F^{MCS}\}^{0.5} = 4\%$. Figure 1 shows a conceptual plot for the MCS samples. These samples center at the origin, the location of the mean value of Z. Since the failure probability is small, most samples are in the non-failure region (SR > 1), while only few samples are in the failure region (SR < 1). Since the c.o.v. of P_F^{MCS} is $\{[1- P(F|\theta)]/N/P(F|\theta)\}^{0.5} \approx [1/(\text{\# of failure samples})]^{0.5}$, the disadvantage of MCS is due to lacking of failure samples.

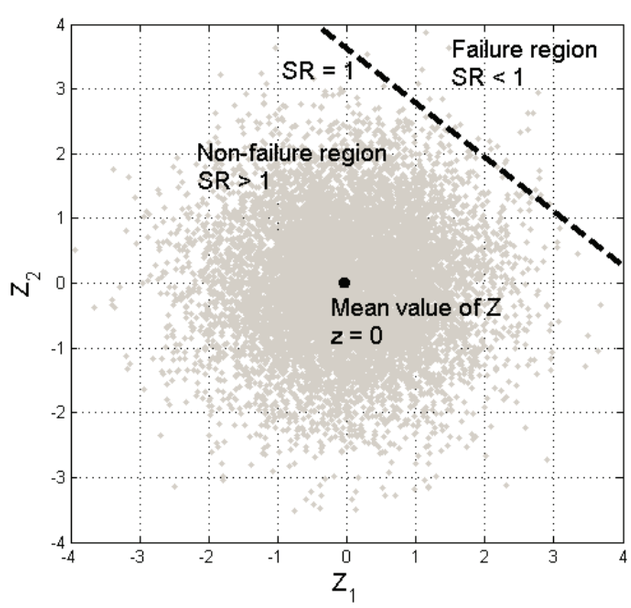

Fig. 1. Conceptual plot for the Monte Carlo samples in the standard Gaussian space

3.2 First-order reliability method

The first-order reliability method (FORM) (Hasofer & Lind, 1974; Der Kiureghian, 2000; Liu and Der Kiureghian, 1991) is not a Monte-Carlo based method. It is introduced herein because it is the most popular reliability method that is used in civil engineering problems and also because the forthcoming Monte-Carlo based method, importance sampling, requires the knowledge of FORM. FORM is based on a mathematical fact that the shortest distance between the limit-state line SR(z,θ) = 1 to the origin z = $0_{p\times1}$ is closely related to P(F | θ) (see Figure 2). In fact, this distance is called the reliability index β, and it can be shown that P(F | θ) is roughly equal to Φ(-β) for relatively simple problems, where Φ is the cumulative density function of standard Gaussian distribution. As a result, determining P(F | θ) is equivalent to finding the shortest distance by the following optimization problem:

$$\min_z \|z\| \qquad \text{subjected to} \quad SR(z,\theta)=1 \tag{13}$$

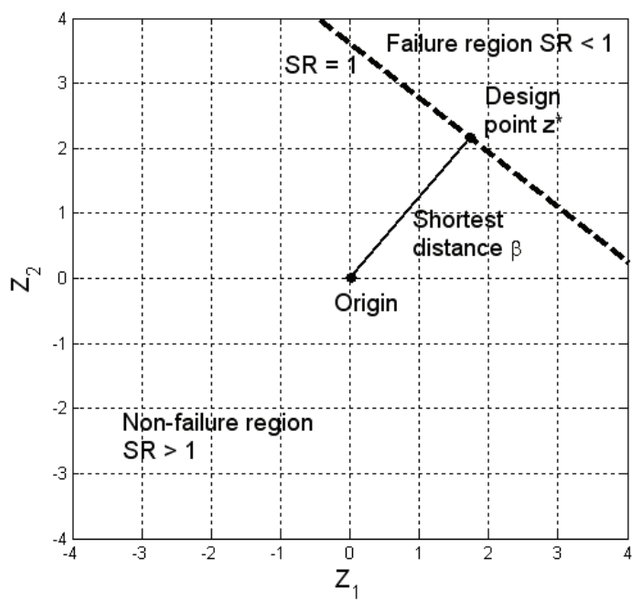

Fig. 2. Significance of the design point in the standard Gaussian space

The solution point of Eq. (13) is called the design point z*. For problems with differentiable SR, the following necessary conditions hold for the design point: (a) SR(z*,θ) = 1 and (b) the gradient vector of SR at z*, i.e., $\nabla_z SR(z^*,\theta)$, is parallel to z*. The reliability index is simply the length of z*. There are many algorithms for finding the design point z*, but the following one is among the simplest (Ang and Tang, 1984):

a. Initialize z_0^* at any location
b. Evaluate $\nabla_z SR\left(z_0^*,\theta\right)=\left[\partial SR\left(z_0^*,\theta\right)/\partial z_1 \quad \partial SR\left(z_0^*,\theta\right)/\partial z_2 \quad \cdots \quad \partial SR\left(z_0^*,\theta\right)/\partial z_p\right]$. This may require numerical approximations for the partial derivatives.
c. Find α_0 such that $SR\left(a_0\cdot\nabla_z SR\left(z_0^*,\theta\right)^T,\theta\right)=1$ and let $z_1^*=a_0\cdot\nabla_z SR\left(z_0^*,\theta\right)^T$. It may require a Newton-method search for determining α_0.

Cycle the Steps b-c until convergence. Once the algorithm converges, it is clear that the converging solution satisfy the two necessary conditions at the same time. For problems with very small failure probability, FORM can be much more efficient than MCS because the former does not require as many SR calculations as the latter. However, for problems with high dimensional Z, the optimization problem in Eq. (13) may become extremely challenging and even become intractable.

For the example design problem of a drilled shaft, the gradient vector is simply

$$\begin{aligned}\nabla_z SR(z,\theta) &= \left[\frac{\partial SR(z,\theta)}{\partial z_{s_u}} \quad \frac{\partial SR(z,\theta)}{\partial z_{\sigma'_{vs}}} \quad \cdots\right] \\ &= \pi B\left[-0.6\times 42.1\times e^{2.7+0.3\left[\ln(70)-0.2z_{s_u}\right]+0.3216z_{S_c}} \quad -0.1\times 7.5\times e^{1.0802-0.6588\ln(45.85)+\left[\ln(350)-0.1z_{\sigma'_{vs}}\right]+0.5414z_{S_s}} \quad \cdots\right]\end{aligned} \tag{14}$$

The above steps are taken to find the design point z*, which is found to satisfy z^*_{su} = -0.245, z^*_{Sc} = -1.333, $z^*_{\sigma'vs}$ = -0.121, z^*_{Ss} = -0.664, $z^*_{\sigma'vg}$ = -0.290, z^*_{Sg} = -1.953, z^*_{qu} = -0.345, z^*_{Sr} = -1.268, z^*_{LD} = 0.644, and z^*_{LL} = 0.894. Note that for all stabilizing variables, the design point coordinates are negative, and for the two destabilizing variables L_D and L_L, the design point coordinates are positive. The distance from the design point to the origin is shortest distance is 3.02, so the estimated $P(F|\theta)$ is equal to $\Phi(-3.02) = 1.3\times10^{-3}$. This result is an approximation to the actual value of $P(F|\theta)$.

3.3 Importance sampling

As mentioned before, for problems with small failure probability, the disadvantage of MCS is that it may require many samples to obtain sufficient failure samples. The importance sampling (IS) (Melchers, 1989; Hohenbichler & Rackwitz, 1988; Der Kureghian & Dakessian, 1998; Au et al., 1999) method mitigates this issue by shifting the standard Gaussian distribution $p(z|\theta)$ to a new center that is closer to the failure region. The most logical choice of this new center is the design point z* from FORM. Let the shifted distribution be $q(z|\theta)$:

$$q(z|\theta) = \frac{1}{2\pi^{p/2}} e^{-\frac{1}{2}(z-z^*)^T(z-z^*)} \tag{15}$$

It is clear that

$$\begin{aligned}P(F|\theta) &= \int 1\left(SR(z,\theta)<1\right)\frac{p(z|\theta)}{q(z|\theta)}q(z|\theta)dz \\ &= \int 1\left(SR(z,\theta)<1\right)\cdot e^{-\frac{1}{2}z^Tz+\frac{1}{2}(z-z^*)^T(z-z^*)}q(z|\theta)dz\end{aligned} \tag{16}$$

According to the Law of Large Number,

$$P(F|\theta) \approx \frac{1}{N}\sum_{i=1}^{N} 1(SR(Z^{(i)},\theta)<1)\cdot e^{\frac{1}{2}z^{*T}z^*-z^{*T}Z^{(i)}} \equiv P_F^{IS} \tag{17}$$

where the samples $Z^{(i)}$ are drawn from the shifted distribution $q(z|\theta)$. Figure 3 shows the conceptual plots for the samples from the IS method: roughly one half of the samples falling

into the failure region. As a result, the c.o.v. for the IS estimator P_F^{IS} can be much smaller than that for P_F^{MCS}.

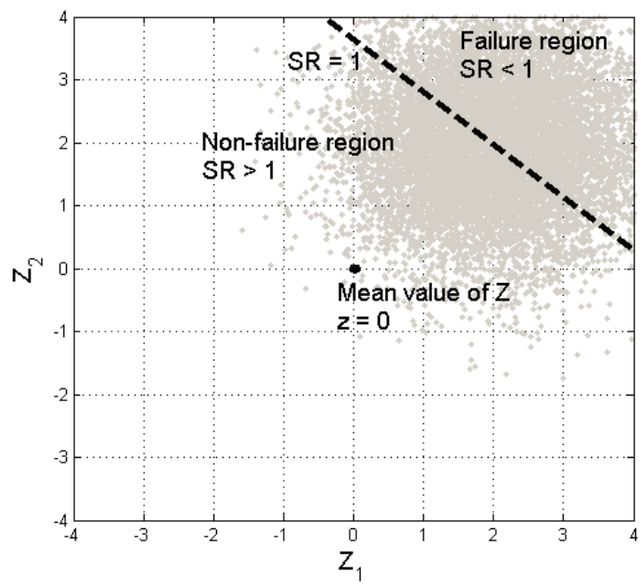

Fig. 3. Conceptual plot for the IS samples in the standard Gaussian space

For the example design problem of a drilled shaft, the following steps can be taken to estimate $P(F|\theta)$:

a. Find the design point z* for FORM.

b. Draw N independent samples of $Z^{(i)} = \{z^{(i)}_{\sigma'vs}, z^{(i)}_{\sigma'vg}, z^{(i)}_{su}, z^{(i)}_{qu}, z^{(i)}_{Sc}, z^{(i)}_{Ss}, z^{(i)}_{Sg}, z^{(i)}_{Sr}, z^{(i)}_{LD}, z^{(i)}_{LL}\}$ from the shifted distribution $q(z|\theta)$.

c. For each sample set, evaluate

$$SR\left(Z^{(i)},\theta\right) = \frac{\pi B\left[\begin{array}{l} e^{2.7+0.3\left[\ln(70)-0.2z^{(i)}_{s_u}\right]+0.3216z^{(i)}_{S_c}}\cdot 42.1 + e^{1.0802-0.6588\ln(45.85)+\left[\ln(350)-0.1z^{(i)}_{\sigma'_{vs}}\right]+0.5414z^{(i)}_{S_s}}\cdot 7.5 \\ +e^{2.1792-0.7528\ln(59.65)+\left[\ln(480)-0.1z^{(i)}_{\sigma'_{vg}}\right]+0.6689z^{(i)}_{S_g}}\cdot 20.1 + e^{3.0253+0.414\left[\ln(900)-0.5z^{(i)}_{q_u}\right]+0.7160z^{(i)}_{S_r}}\cdot 5.0 \end{array}\right]}{e^{\ln\left(8000/\sqrt{1+0.1^2}\right)+\sqrt{\ln\left(1+0.1^2\right)}\cdot z^{(i)}_{LD}} + e^{\ln\left(4000/\sqrt{1+0.25^2}\right)+\sqrt{\ln\left(1+0.25^2\right)}\cdot z^{(i)}_{LL}}} \quad (18)$$

d. Let

$$P_F^{IS} = \frac{1}{N}\sum_{i=1}^{N} 1(SR(Z^{(i)},\theta)<1)\cdot e^{\frac{1}{2}z^{*T}z^{*}-z^{*T}Z^{(i)}} \quad (19)$$

By taking N = 1000, P_F^{IS} is found to be around 6.1×10^{-4}. Its c.o.v. is estimated to be around 7%. Compared to MCS using 10^6 samples yielding a 4% c.o.v., the IS method is much more efficient. It seems like the IS method improves MCS, but in fact this is not entirely true: as reported in Au & Beck (2003), the IS method may suffer from the issue of high Z dimension (as FORM does), but MCS does not have such limitation.

3.4 Subset simulation

Among the previous reliability methods, no method is suitable for complex problems with dimensional Z and with small failure probability. FORM is suitable for problems with small failure probability but not for those with high dimensional Z. In contrast, MCS is robust with Z dimension and problem complexity but may be inefficient for problems with small failure probability. The IS method also suffers from the issue of high dimensional Z.

Subset simulation (Subsim) (Au & Beck, 2001) is among the few reliability methods that are robust again all the aforementioned aspects. Subsim inherits most advantages of MCS: it is robust against Z dimension and problem complexity, but its computational cost for problems with small failure probability is typically acceptable. The basic idea of Subsim is to express the failure probability $P(F|\theta)$ as a product of several larger conditional probabilities, so that the estimation of $P(F|\theta)$ can be achieved by estimating the conditional probabilities and multiply them together.

Let us first introduce intermediate failure events $\{F_1,F_2,\ldots,F_m\}$. For our purpose, these failure events can be defined as

$$F_i = \{SR(z,\theta) < b_i\} \quad i = 1,\ldots,m \tag{20}$$

where $b_1 > b_2 > \ldots > b_m = 1$. It is then clear that the intermediate failure events are nested, i.e., $F_1 \supset F_2 \supset \ldots \supset F_m = F$. Moreover, the failure event F is the intersection of all intermediate failure events. According to the operation of conditional probability,

$$\begin{aligned} P(F|\theta) &= P\left(\bigcap_{i=1}^{m} F_i \,|\, \theta\right) = P\left(F_m \,\middle|\, \bigcap_{i=1}^{m-1} F_i, \theta\right) \cdot\cdots\cdot P(F_2|F_1,\theta)\cdot P(F_1|\theta) \\ &= P(F_m|F_{m-1},\theta)\cdot\cdots\cdot P(F_2|F_1,\theta)\cdot\cdot P(F_1|\theta) = \prod_{i=2}^{m} P(F_i|F_{i-1},\theta)\cdot P(F_1|\theta) \end{aligned} \tag{21}$$

As a result, the estimation of $P(F|\theta)$ can be achieved through the estimation of the conditional probabilities $P(F_1|\theta)$, $P(F_2|F_1,\theta)$, …, $P(F_m|F_{m-1},\theta)$. Note that although $P(F|\theta)$ may be very small, the conditional probabilities $P(F_1|\theta)$, …, $P(F_m|F_{m-1},\theta)$ can be made large and can be estimated in a more accurate manner. Hence, the issue of small failure probability for MCS is resolved. In the following, the estimation of these conditional probabilities will be addressed.

Estimation of $P(F_1|\theta)$

This estimation can be easily done by using MCS, i.e., draw N_0 samples of Z from the standard Gaussian distribution, denoted by $\{Z_0^{(k)}: k=1,\ldots, N_0\}$. Then,

$$P(F_1|\theta) \approx \frac{1}{N_0}\sum_{i=1}^{N_0} 1(SR(Z_0^{(k)}) < b_1) \equiv P_1^{SS} \tag{22}$$

Note that $P(F_1|\theta)$ is typically quite large, hence the c.o.v. of P_1^{SS} is typically small. Among the N_0 samples, let there be R_0 samples, denoted by $\{Z_0^{*(k)}: k=1, \ldots, R_0\}$, satisfying $SR < b_1$. Let us call these samples the below-b_1 samples. These samples are actually distributed as $p(z|F_1,\theta)$, which can be expressed as

$$p(z \mid F_1,\theta) = \frac{p(z \mid \theta)\cdot 1(SR(z,\theta) < b_1)}{P(F_1 \mid \theta)} = \frac{e^{\frac{-1}{2}z^T z}\cdot 1(SR(z,\theta) < b_1)}{(2\pi)^{p/2}\, P(F_1 \mid \theta)} \tag{23}$$

Estimation of $P(F_2 \mid F_1,\theta)$

Suppose we know how to draw N_1 samples, denoted by $\{Z_1^{(k)}: k=1,\ldots, N_1\}$, from $p(z \mid F_1,\theta)$. The estimation of $P(F_2 \mid F_1,\theta)$ can then be made:

$$P(F_2 \mid F_1,\theta) \approx \frac{1}{N_1}\sum_{k=1}^{N_1} 1(SR(Z_1^{(k)},\theta) < b_2) \equiv P_2^{SS} \tag{24}$$

However, drawing samples from $p(z \mid F_1,\theta)$ is nontrivial. Recall that the below-b_1 samples $\{Z_0^{*(k)}: k=1, \ldots, R_0\}$ from stage 1 are already distributed as $p(z \mid F_1,\theta)$. It is then possible to use the Metropolis algorithm (Au & Beck, 2001) to generate more samples that are also distributed as $p(z \mid F_1,\theta)$. Each below-b_1 sample $Z_0^{*(k)}$ is taken to be the initial sample of a Markov chain whose stationary distribution is $p(z \mid F_1,\theta)$. Let further the j-th sample of the k-th Markov chain be $Z_1^{(k,j)}$. The k-th below-b_1 sample $Z_0^{*(k)}$ from stage 1 is therefore $Z_1^{(k,1)}$. The following Metropolis algorithm can then be taken to generate the rest samples $\{Z_1^{(k,j)}: j = 2, \ldots\}$ for the k-th Markov chain:

a. Given the j-th sample $Z_1^{(k,j)}$ in this chain, draw a candidate sample Z_1^C from a Gaussian distribution centered at $Z_1^{(k,j)}$ and with a chosen covariance matrix Σ.
b. Compute the following ratio r:

$$r = \frac{p\left(Z_1^C \mid F_1,\theta\right)}{p\left(Z_1^{(k,j)} \mid F_1,\theta\right)} = e^{\frac{-1}{2}Z_1^{CT}Z_1^C + \frac{1}{2}Z_1^{(k,j)T}Z_1^{(k,j)}} \cdot \frac{1\left[SR\left(Z_1^C,\theta\right) < b_1\right]}{1\left[SR\left(Z_1^{(k,j)},\theta\right) < b_1\right]} \tag{25}$$

c. Accept the candidate sample, i.e., let $Z_1^{(k,j+1)}$ be Z_1^C, with probability min(1,r), and repeat the previous sample, i.e., let $Z_1^{(k,j+1)}$ be $Z_1^{(k,j)}$, with probability 1-min(1,r).

Suppose each Markov chain generate M_1 samples, the following samples $\{Z_1^{(k,j)}: k = 1,\ldots,R_0, j = 1, \ldots, M_1\}$ are available. All these samples are distributed as $p(z \mid F_1,\theta)$, so there are $N_1 = R_0M_1$ samples, rearranged to be $\{Z_1^{(k)}: k = 1,\ldots, N_1\}$, for the estimation of $P(F_2 \mid F_1,\theta)$ in Eq. (24). Among these N_1 samples, let there be R_1 samples, denoted by $\{Z_1^{*(i)}: i=1, \ldots, R_1\}$, satisfying SR < b_2. These samples are the below-b_2 samples and are actually distributed as $p(z \mid F_2,\theta)$, and the same Metropolis algorithm can be used to generate more samples from $p(z \mid F_2,\theta)$ to estimate $P(F_3 \mid F_2,\theta)$. This process continues until $P(F_m \mid F_{m-1},\theta)$ is estimated. Finally, $P(F \mid \theta)$ can be estimated as

$$P(F \mid \theta) \approx \prod_{i=1}^{m} P_i^{SS} = \prod_{i=1}^{m} \frac{R_{i-1}}{N_{i-1}} \equiv P_F^{SS} \tag{26}$$

In real application of Subsim, the threshold b_i is adaptively chosen so that $R_{i-1} = N_{i-1}/10$ (except the final stage R_{m-1} is not equal to $N_{m-1}/10$), i.e., b_i is taken to be the 10% percentile of $\{SR(Z_{i-1}^{(k)},\theta): k = 1, \ldots, N_{i-1}\}$. Moreover, each Markov chain generate 10 samples, including the below-b_{i-1} sample from the previous stage. This makes $N_0 = N_1 = \ldots = N_{m-1} = N_{SS}$. Au and Beck (2001) show that the estimator P_F^{SS} is asymptotically unbiased. Ching et al. (2005) show that the c.o.v. of P_F^{SS} $[\delta(P_F^{SS})]$ is bounded by

$$\sqrt{\frac{1}{N_{SS}}\left(9(m-1)+\frac{1-P_m^{SS}}{P_m^{SS}}\right)} \le \delta\left(P_F^{SS}\right) \le \sqrt{\frac{1}{N_{SS}}\left(9+99(m-2)+\frac{1-P_m^{SS}}{P_m^{SS}}\cdot 11\right)} \tag{27}$$

For the example design problem of a drilled shaft, 1000 MCS samples of Z (N_{SS} = 1000) are draw from the standard Gaussian distribution for the first stage. For each sample $Z^{(i)}$, its SR sample value is evaluated to get $SR(Z^{(i)},\theta)$. The leftmost plot in Figure 4 shows such SR samples. It is clear that there are no failure samples, i.e., samples satisfying SR < 1. The first threshold SR = b_1, shown in the left plot as the horizontal line, is then identified as the 10% percentile of the SR sample values, and F_1 event is therefore $\{SR(z,\theta) < b_1\}$. As a result, $P(F_1|\theta) \approx P_1^{SS} = 0.1$. The 100 below-$b_1$ samples (the darker dots) are distributed as $p(z|F_1,\theta)$. These below-b_1 samples are then taken in the Metropolis algorithm to generate more samples also distributed as $p(z|F_1,\theta)$: each below-b_1 sample is taken to lead a Markov chain that generated 9 more samples distributed as $p(z|F_1,\theta)$. These 1000 new samples are seen in the middle left plot. Note that all these samples have SR values less than b_1 because they are distributed as $p(z|F_1,\theta)$.

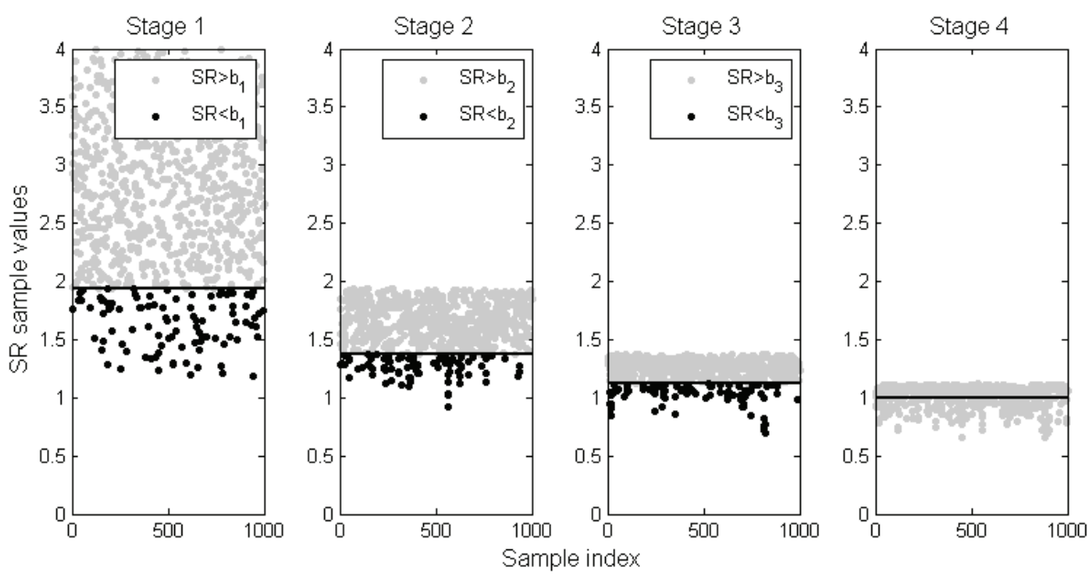

Fig. 4. Evolution of the SR samples in various stages (stage 1 to 4 from left to right) for Subsim

The second threshold SR = b_2, shown in the middle left plot as the horizontal line, is then identified as the 10% percentile of the SR sample values, and F_2 event is therefore $\{SR(Z,\theta) < b_2\}$. As a result, $P(F_2|F_1,\theta) \approx P_2^{SS} = 0.1$. Similarly, the 100 below-b_2 samples (the darker dots) are distributed as $p(z|F_2,\theta)$. These below-b_2 samples are then taken in the Metropolis algorithm to generate 1000 samples also distributed as $p(z|F_2,\theta)$, i.e., the samples seen in the rightmost plot.

The third stage is similar to the previous stages (see the middle right plot). Similarly, the threshold b_3 is adaptively chosen, and $P(F_3|F_2,\theta) \approx P_3^{SS} = 0.1$. The fourth stage is somewhat different because now the SR values of the 1000 samples distributed as $p(z|F_3,\theta)$ (the gray dots in the rightmost plot) are close to the failure threshold b = 1. The 10% percentile of the

SR values is found to be less than 1, i.e., $P(F|F_3,\theta) > 0.1$. In fact, 28.9% of 1000 SR values are less than 1. In this scenario, the fourth threshold b_4 is no longer adaptively chosen as the 10% percentile but is taken to be 1, and the entire Subsim algorithm ends at this stage. Consequently, $P(F_4|F_3,\theta) = P(F|F_3,\theta) \approx P_4^{SS} = 0.289$, and the Subsim estimate for $P(F|\theta)$ is simply $P_F^{SS} = P_1^{SS}\times P_2^{SS}\times P_3^{SS}\times P_4^{SS} = 2.89\times10^{-4}$. The bounds for this estimator can be found to be

$$\sqrt{\frac{1}{1000}\left(9\times3+\frac{1-0.289}{0.289}\right)} = 17.2\% \le \delta\left(P_F^{SS}\right) \le 48.4\% = \sqrt{\frac{1}{1000}\left(9+99\times2+\frac{1-0.289}{0.289}\cdot11\right)} \tag{28}$$

4. Reliability-based design

In this section, state-of-the-art developments in the Monte Carlo based reliability-based design methods are reviewed. Reliability-based design (RBD) is the inverse problem of reliability analysis: the purpose of reliability analysis is to obtain the reliability given the design dimension of an engineering system, but the purpose of RBD is to design for the dimension that provides the target reliability. The Monte Carlo based methods recently developed by Ching & Phoon (2010) will be introduced. These Monte Carlo based methods are able to convert the RBD design constraint into simple algebraic design equations. Moreover, these methods inherit most of the advantages of MCS, i.e., not limited by the Z dimension, problem complexity, etc. One limitation is that the number of design parameters, e.g., dimension of θ, cannot be too large, which is usually the case for geotechnical designs.
For RBD, the objective is to enforce the following *probabilistic* constraint during the design process:

$$P\left(SR(Z,\theta)<1\middle|\theta\right) = \int p(z|\theta)\cdot 1\left(SR(z,\theta)<1\right)dz \le P_F^* \tag{29}$$

where P_F^* is the target failure probability; 1(.) is the indicator function, i.e. it is equal to 1 if the argument (safety ratio less than one) is true; otherwise, it is zero. The purpose of this section is to show that this probabilistic design constraint can be transformed into a deterministic *algebraic* constraint of the following format:

$$c(\theta) \ge 1 \tag{30}$$

Intuitively, $c(\theta)$ should be taken to be a conservative version of $SR(z^*,\theta)$, i.e., $c(\theta) < SR(z^*,\theta)$, where z^* is the characteristic value of Z, which can be taken to be the mean value or median value of Z. As a result, requiring $c(\theta) \ge 1$ is much stronger than requiring $SR(z^*,\theta) \ge 1$, leading to a more conservative design with a small target failure probability P_F^*. Depending on how this conservatism is applied, there are four possible algebraic design formats:

a. Safety factor design
   For this design format, $c(\theta)$ is taken to be the η quantile (η is called the probability threshold) of $SR(Z,\theta)$, denoted by $SR^{\eta}(\theta)$. The probability threshold η is typically taken to be a small number, e.g., 0.05, to ensure a conservative design. For a normally distributed SR, the 5% quantile, $SR^{0.05} = \mu_{SR}\,(1 - 1.645\,\delta_{SR})$, in which μ_{SR} and δ_{SR} are the mean and c.o.v. of SR. This definition is sensible because we are applying a value less than the mean. Requiring $c(\theta) \ge 1$ is equivalent to requiring

$$c(\theta)=\frac{SR\left(z^{*},\theta\right)}{SR\left(z^{*},\theta\right)/c(\theta)}=\frac{SR\left(z^{*},\theta\right)}{SR\left(z^{*},\theta\right)/SR^{\eta}(\theta)}=\frac{SR\left(z^{*},\theta\right)}{SF^{\eta}}\geq 1 \tag{31}$$

where SF is the safety factor, which clearly depends on η. Note that the required SF is simply the nominal safety ratio $SR(z^{*},\theta)$ divided by the η quantile $SR^{\eta}(\theta)$.

b. Load-resistance factor design (LRFD)

For many geotechnical design problems, the safety ratio $SR(z,\theta)$ has the format of

$$SR(z,\theta)=\frac{S(z,\theta)}{L_D(z,\theta)+L_L(z,\theta)} \tag{32}$$

where S is the total resistance, and L_D and L_L are the dead and live loads. In the case, a possible choice for $c(\theta)$ is

$$c(\theta)=\frac{S^{\eta}(\theta)}{L_D^{1-\eta}(\theta)+L_L^{1-\eta}(\theta)} \tag{33}$$

where $S^{\eta}(\theta)$ is the η quantile of $S(Z,\theta)$, and $L_D^{1-\eta}(\theta)$ and $L_L^{1-\eta}(\theta)$ are the 1-η quantiles of $L_D(Z,\theta)$ and $L_L(Z,\theta)$. The same probability threshold η (e.g., 0.05) is applied to all three random variables. This would not only ensure a conservative design but also ensure that all random variables have the same exceedance/non-exceedance probability over the corresponding quantiles. Requiring $c(\theta)\geq 1$ is equivalent to requiring

$$c(\theta)=\frac{\dfrac{S^{\eta}(\theta)}{S\left(z^{*},\theta\right)}S\left(z^{*},\theta\right)}{\dfrac{L_D^{1-\eta}(\theta)}{L_D\left(z^{*},\theta\right)}L_D\left(z^{*},\theta\right)+\dfrac{L_L^{1-\eta}(\theta)}{L_L\left(z^{*},\theta\right)}L_L\left(z^{*},\theta\right)}=\frac{\gamma_S^{\eta}\cdot S\left(z^{*},\theta\right)}{\gamma_{LD}^{\eta}\cdot L_D\left(z^{*},\theta\right)+\gamma_{LL}^{\eta}\cdot L_L\left(z^{*},\theta\right)}\geq 1 \tag{34}$$

where γ_S is the resistance factor, while γ_{LD} and γ_{LL} are the load factors. These factors clearly depend on η. It is also clear that $\gamma_S < 1$ and that $\gamma_{LD},\gamma_{LL} > 1$ if the probability threshold η is small. Note that the required resistance factor is simply the η quantile $S^{\eta}(\theta)$ divided by the nominal resistance $S(z^{*},\theta)$, and the required load factor is simply the 1-η quantile $L^{1-\eta}(\theta)$ divided by the nominal load $L(z^{*},\theta)$.

c. Multiple resistance factor design (MRFD)

For some geotechnical design problems, the total resistance S is contributed by several different components. Let us denote the various components by S_x. For the drilled shaft example, the subscript 'x' can be either 'c', 's', 'g', or 'r', depending on which stratum provides the side resistance, and

$$SR(z,\theta)=\frac{S_c(z,\theta)+S_s(z,\theta)+S_g(z,\theta)+S_r(z,\theta)}{L_D(z,\theta)+L_L(z,\theta)} \tag{35}$$

In the case, a possible choice for $c(\theta)$ is

$$c(\theta)=\frac{S_c^{\eta}(\theta)+S_s^{\eta}(\theta)+S_g^{\eta}(\theta)+S_r^{\eta}(\theta)}{L_D^{1-\eta}(\theta)+L_L^{1-\eta}(\theta)} \tag{36}$$

Again, the same probability threshold η is applied to all six random variables to ensure that all random variables have the same exceedance/non-exceedance probability over the corresponding quantiles. Requiring c(θ) ≥ 1 is equivalent to requiring

$$c(\theta)=\frac{\frac{S_c^{\eta}(\theta)}{S_c(z^*,\theta)}S_c(z^*,\theta)+\ldots+\frac{S_r^{\eta}(\theta)}{S_r(z^*,\theta)}S_r(z^*,\theta)}{\frac{L_D^{1-\eta}(\theta)}{L_D(z^*,\theta)}L_D(z^*,\theta)+\frac{L_L^{1-\eta}(\theta)}{L_L(z^*,\theta)}L_L(z^*,\theta)}=\frac{\gamma_{Sc}^{\eta}\cdot S_c(z^*,\theta)+\ldots+\gamma_{Sr}^{\eta}\cdot S_r(z^*,\theta)}{\gamma_{LD}^{\eta}\cdot L_D(z^*,\theta)+\gamma_{LL}^{\eta}\cdot L_L(z^*,\theta)}\geq 1 \tag{37}$$

where γ_{Sc}, γ_{Ss}, γ_{Sg}, and γ_{Sr} are the resistance factors, while γ_{LD} and γ_{LL} are the load factors. It is clear that all resistance factors are less than 1 and that $\gamma_{LD},\gamma_{LL} > 1$ if the probability threshold η is small. Note that the required resistance factor is simply the η quantile $S^{\eta}(\theta)$ divided by the nominal resistance $S(z^*,\theta)$, and the required load factor is simply the 1-η quantile $L^{\eta}(\theta)$ divided by the nominal load $L(z^*,\theta)$.

d. Partial factor design

In contrast to LRFD and MRFD where the factors are applied to load and resistance terms, the partial-factor design format applies the (partial) factors to Z directly, i.e.,

$$c(\theta)=SR\left(z_1^{\eta},z_2^{\eta},\ldots,z_p^{1-\eta},\theta\right)\geq 1 \tag{38}$$

where either the η or 1-η quantile of Z_i is taken depending on its characteristic. For Z_i that is clearly stabilizing, η quantile of Z_i should be taken, while 1-η quantile should be adopted for destabilizing Z_i. For Z_i that is not influential or whose effect cannot be clearly discerned as stabilizing or destabilizing, the mean or median value may be taken. Requiring c(θ) ≥ 1 is equivalent to requiring

$$c(\theta)=SR\left(\frac{z_1^{\eta}}{z_1^*}z_1^*,\frac{z_2^{\eta}}{z_2^*}z_2^*,\ldots,\frac{z_p^{1-\eta}}{z_p^*}z_p^*,\theta\right)=SR\left(\gamma_1^{\eta}z_1^*,\gamma_2^{\eta}z_2^*,\ldots,\gamma_p^{\eta}z_p^*,\theta\right)\geq 1 \tag{39}$$

where γ's are the partial factors. For stabilizing random variables, the partial factors are less than 1, and for destabilizing random variables, the partial factors are greater than 1. Note that the required partial factor is simply the η quantile z_i^{η} divided by its characteristic value z_i^* if z_i is stabilizing and is the 1-η quantile $z_i^{1-\eta}$ divided by its characteristic value z_i^* if z_i is destabilizing.

Note that algebraic design constraints based on the above four formats are convenient, because to make sure c(θ) ≥ 1 holds, it only requires a *single* algebraic evaluation of SR(Z,θ) using deterministic factors γ's and characteristic values z^*. However, RBD is more theoretically involved and computationally demanding: in order to verify if Eq. (29) holds, it requires a reliability analysis, which in the most general case, may take millions of Monte Carlo evaluations of SR(Z,θ). If the equivalence between the four algebraic design formats and the RBD can be established, it will be significant in the following practical sense:

a. One can then achieve a RBD by using any of the four algebraic formats, which is much simpler and more convenient than the former.
b. Practical geotechnical engineers who are not familiar with reliability concept can easily achieve reliability-based design by using the equivalence.

To establish the equivalence, it is necessary to find the relation between the probability threshold η and the target failure probability P_F^*. It is clear that η controls the degree of conservatism. Recall that the algebraic design constraint is to require $c(\theta) \geq 1$, hence a smaller η will lead to a more conservative design because the value of $c(\theta)$ decreases with decreasing η. As a result, it seems reasonable to use a small η when the target failure probability P_F^* is small, and vice versa.

4.1 Statement of the equivalence principle

In Ching & Phoon (2010), it is postulated that the four algebraic design formats associated with a proper probability threshold η can be made equivalent to rigorous RBD based on a direct probability check. The key hypothesis needed for the principle to be practical is also clarified explicitly using a mathematical proof in Appendix. To be specific, we postulate that there exists pairs of (η, P_F^*) such that the following constraints are equivalent:

$$c(\theta) \geq 1 \tag{40}$$

and

$$P\left(SR(Z,\theta) < 1 \middle| \theta\right) \leq P_F^* \tag{41}$$

Moreover, the functional relation between the pair (η, P_F^*) is as follows:

$$P\left(\frac{c(\theta)}{SR(Z,\theta)} > 1 \middle| \theta\right) = P_F^* \tag{42}$$

In Eq. (42), note that the numerator is a deterministic number and the denominator is a random variable depending on θ. Equation (42) is the key equation in the proposed algebraic design formats.

4.2 Uniformity of the equivalence

The proposed approach is not practical if the relation between (η, P_F^*) depends on the design parameter θ. If this happens, one needs to find the (η, P_F^*) relation for all design scenarios under consideration, and the resulting design factors will vary for different design scenarios. In principle, the distribution $c(\theta)/SR(Z,\theta)$ should depend on θ and hence, we state the contrary as a hypothesis in Appendix. The empirical study shows that this hypothesis is reasonable. In this section, we attempt to explain qualitatively why the distribution of $c(\theta)/SR(Z,\theta)$ does not appear to change drastically with θ.

This weak dependency is explained as follows by considering the special case of $\eta = 0.5$. In this case, $c(\theta)$ is similar to the nominal value of $SR(Z,\theta)$. Although the distribution of $SR(Z,\theta)$ may change drastically with θ (see Figure 5(a)), the distribution of $c(\theta)/SR(Z,\theta)$ usually does not (see Figure 5(b)) due to the cancellation effect between $SR(Z,\theta)$ and the nominal value of $SR(Z,\theta)$. The same phenomenon remains for $\eta \neq 0.5$. Later in the demonstrating drilled shaft example, the invariance of the $c(\theta)/SR(Z,\theta)$ distribution over θ will be verified empirically.

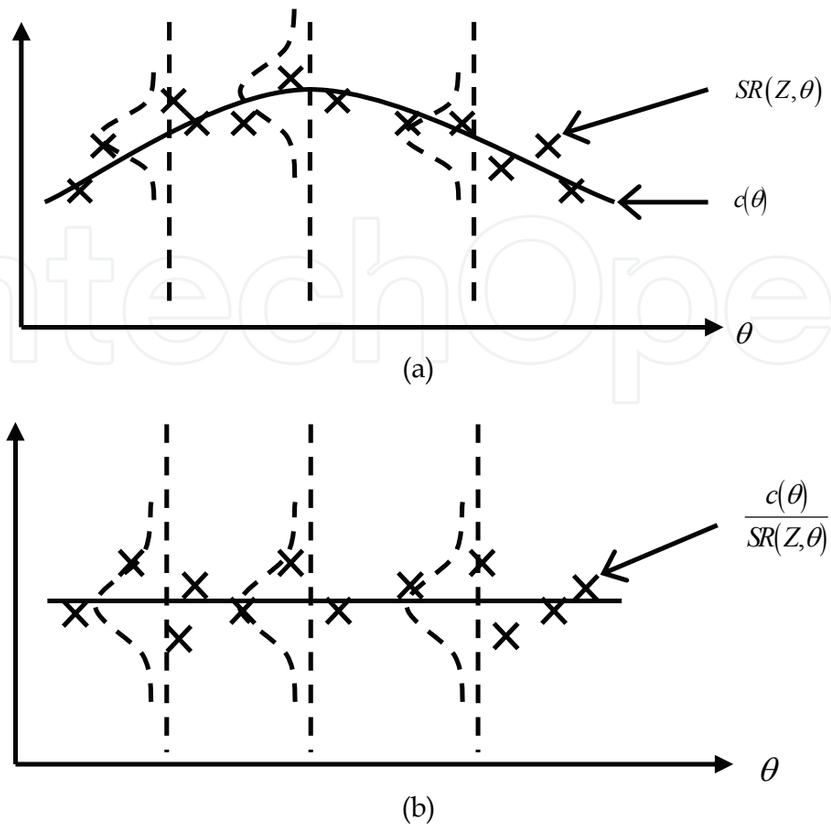

Fig. 5. Illustration of the distributions of SR(Z,θ) and c(θ)/SR(Z,θ) (a). Illustration of the distribution of SR(Z,θ) (b). Illustration of the distribution of c(θ)/SR(Z,θ)

If the distribution of c(θ)/SR(Z,θ) is indeed approximately invariant over θ, the relation between (η, P_F^*) can be found by the following equation:

$$P\left(\frac{c(\theta)}{SR(Z,\theta)} > 1\right) = P_F^* \tag{43}$$

where θ is treated as random and uniformly distributed over the allowable design region. It is clear that P_F^* is simply the exceedance probability of c(θ)/SR(Z,θ) over the unity. Therefore, the relation between η and P_F^* can be determined by any reliability method, in particular the Monte Carlo simulation (MCS): draw N samples of (Z,θ), where Z samples are drawn from p(z|θ), and θ samples are drawn from the uniform distribution over the allowable design region. Each (Z,θ) sample pair can be used to obtain a sample of c(θ)/SR(Z,θ). At the end of MCS, we have N samples of c(θ)/SR(Z,θ). For a chosen η value, the corresponding P_F^* value can be simply estimated as the ratio that c(θ)/SR(Z,θ) samples are greater 1. By changing the η value and repeating the MCS, one can obtain the entire relation between η and P_F^*

According to Appendix and the above discussions, the reliability constraint Eq. (41) can be transformed into the algebraic design constraint Eq. (40) if the distribution of c(θ)/SR(Z,θ) is invariant over θ. Let us denote $\Sigma_R = \{\theta: P(SR(Z,\theta)<1 \mid \theta) \le P_F^*\}$ be the design region that satisfies the reliability constraint that failure probability is less or equal to the target failure

probability, P_F^*. From Appendix, if the distribution of $c(\theta)/SR(Z,\theta)$ is indeed invariant over θ, it is assured that the region Σ_R is identical to the following region: $\Sigma_S = \{\theta: c(\theta)\geq 1\}$, so the algebraic design formats and RBD become equivalent.

4.3 Determining the design factors based on the principle

The following steps can be taken to find the design factors, such as safety factor, load factors, resistance factors, and partial factors, corresponding to any prescribed P_F^*:

1. Find the relation between η and the design factors:
   a. For the safety-factor design format, the safety factor SF^η is $SR(z^*,\theta)/SR^\eta(\theta)$, where the η quantile $SR^\eta(\theta)$ can be easily estimated by MCS.
   b. For LRFD, the resistance factor γ_S^η is $S^\eta(\theta)/S(z^*,\theta)$, while the load factor γ_{Lx}^η is $L_x^{1-\eta}(\theta)/L_x(z^*,\theta)$.
   c. For MRFD, the resistance factor γ_{Sx}^η is $S_x^\eta(\theta)/S_x(z^*,\theta)$, while the load factor γ_{Lx}^η is $L_x^{1-\eta}(\theta)/L_x(z^*,\theta)$.
   d. For the partial-factor design format, the partial factor γ_i^η for Z_i is z_i^η/z_i^* if Z_i is stabilizing and is $z_i^{1-\eta}/z_i^*$ if Z_i is destabilizing.
2. Find the relation between the pair (η,P_F^*) by solving Eq. (43). This has been presented previously, i.e., simulating $c(\theta)/SR(Z,\theta)$ samples and find the ratio of less than 1. In Eq. (43), the definition of $c(\theta)$ for various algebraic design formats are different:
   a. For the safety-factor design format, $c(\theta) = SR(z^*,\theta)/SF^\eta$.
   b. For LRFD, $c(\theta) = \gamma_S^\eta S(z^*,\theta)/[\gamma_{LD}^\eta L_D(z^*,\theta) + \gamma_{LL}^\eta L_L(z^*,\theta)]$.
   c. For MRFD, $c(\theta) = \Sigma_x[\gamma_{Sx}^\eta S_x(z^*,\theta)]/[\gamma_{LD}^\eta L_D(z^*,\theta) + \gamma_{LL}^\eta L_L(z^*,\theta)]$.
   d. For the partial-factor design format, $c(\theta) = SR(\gamma_1^\eta z_1^*,\gamma_2^\eta z_2^*,\ldots, \gamma_p^\eta z_p^*,\theta)$.
3. Given the prescribed target failure probability, P_F^*, find the corresponding probability threshold η from the result in Step 1.
4. Once the corresponding probability threshold η is found, the required design factor can be determined accordingly according to the relations presented in Step 1.
   a. For safety-factor design format, the resulting algebraic design constraint is $c(\theta) = SR(z^*,\theta)/SF^\eta \geq 1$.
   b. For LRFD, the resulting algebraic design constraint is $c(\theta) = \gamma_S^\eta S(z^*,\theta)/[\gamma_{LD}^\eta L_D(z^*,\theta)+\gamma_{LL}^\eta L_L(z^*,\theta)] \geq 1$.
   c. For MRFD, the resulting algebraic design constraint is $c(\theta) = \Sigma_x[\gamma_{Sx}^\eta S_x(z^*,\theta)]/[\gamma_{LD}^\eta L_D(z^*,\theta)+\gamma_{LL}^\eta L_L(z^*,\theta)] \geq 1$.
   d. For partial-factor design format, the resulting algebraic design constraint is $c(\theta) = SR(\gamma_1^\eta z_1^*,\gamma_2^\eta z_2^*,\ldots, \gamma_p^\eta z_p^*,\theta) \geq 1$.

According to the derivations given in Appendix A, the design based on the algebraic constraint $c(\theta) \geq 1$ is identical to the probabilistic constraint $P(SR(Z,\theta)<1 \mid \theta) \leq P_F^*$.

4.4 Example

The same drilled shaft problem that was taken in the previous section will be used to demonstrate the RBD. Let the diameter B and length L be the two design parameters (i.e., θ contains B and L) that are subjected to change in the design process. Let us further assume there is a practicality design constraints 0.8 m $\leq$ B $\leq$ 1.5 m and 49.6 m $\leq$ L $\leq$ 74.7 m. These practicality design constraints are realistic since most drilled shafts have diameters ranging from 0.8 m to 1.5 m and since most drilled shafts may be bottomed in strata with high strengths, in our case, the gravel or rock layer (49.6 m and 74.7 m are the limiting depths of

the gravel and rock layers). Other conditions, such as the ground profile and the mean values and c.o.v.s of L_D and L_L, remain the same as in the reliability analysis section. The target failure probability P_F^* is taken to be 0.001, i.e., the design goal is to adopt a certain combination of B and L so that $P(SR(Z,\theta)<1 \mid \theta) \le 0.001$. For brevity, only the detailed steps for MRFD will be demonstrated, but the results for the safety-factor design, LRFD, and partial-factor design will be still presented. Recall that the total resistance S is provided by S_c, S_s, S_g, and S_r:

$$\begin{aligned}
S_c(Z,\theta) &= \pi B e^{2.7+0.3\left[\ln s_{u,m}-e_{s_u}\right]+\varepsilon_{S_c}} t_c \\
S_s(Z,\theta) &= \pi B e^{1.0802-0.6588\ln(d_s)+\left[\ln(\sigma'_{vs,m})-e_{\sigma'_{vs}}\right]+\varepsilon_{S_s}} t_s \\
S_g(Z,\theta) &= \pi B e^{2.1792-0.7528\ln(d_g)+\left[\ln(\sigma'_{vg,m})-e_{\sigma'_{vg}}\right]+\varepsilon_{S_g}} \min\left(L-t_c-t_s,t_g\right) \\
S_r(Z,\theta) &= \pi B e^{3.0253+0.414\left[\ln q_{u,m}-e_{q_u}\right]+\varepsilon_{S_r}} \min\left(L-t_c-t_s-t_g,0\right)
\end{aligned} \tag{44}$$

Note that B and L are now subjected to change in the design process; min(A,B) is the minimum value among A and B, and the two terms $\min(L-t_c-t_s, t_g)$ and $\min(L-t_c-t_s-t_g, 0)$ are there due to the fact that the shaft may not penetrate the entire gravel and rock layers when the length L is not large enough.

Step 1 – determine the relations between η and the MRFD design factors. Note that these relations are independent of the diameter B and the embedment lengths in various strata, i.e. t_c, t_s, $\min(L-t_c-t_s, t_g)$, etc. Therefore, these relations can be determined by fixing B and L at any values, for this example, B = 1.2 m and L = 74.7 m. Recall that the resistance factor corresponding to η is $S_x^{\eta}(\theta)/S_x(z^*,\theta)$, while the load factor corresponding to η is $L_x^{1-\eta}(\theta)/L_x(z^*,\theta)$. The nominal values S_x^* and those for the dead and live loads are taken to be

$$\begin{aligned}
S_c^* &= S_c\left(z^*,\theta\right) = \pi B e^{2.7+0.3\ln s_{u,m}} t_c \\
S_s^* &= S_s\left(z^*,\theta\right) = \pi B e^{1.0802-0.6588\ln(d_s)+\ln(\sigma'_{vs,m})} t_s \\
S_g^* &= S_g\left(z^*,\theta\right) = \pi B e^{2.1792-0.7528\ln(d_g)+\ln(\sigma'_{vg,m})} \min\left(L-t_c-t_s,t_g\right) \\
S_r^* &= S_r\left(z^*,\theta\right) = \pi B e^{3.0253+0.414\ln q_{u,m}} \min\left(L-t_c-t_s-t_g,0\right) \\
L_D^* &= \mu_{LD} = 8000kN \qquad L_L^* = \mu_{LL} = 4000kN
\end{aligned} \tag{45}$$

As a result,

$$\begin{aligned}
\gamma_{Sc}^{\eta} &= \left[\eta \text{ quantile of } S_c(Z,\theta)\right] / S_c\left(z^*,\theta\right) = \eta \text{ quantile of } \left[S_c(Z,\theta)/S_c\left(z^*,\theta\right)\right] \\
&= \eta \text{ quantile of } e^{\varepsilon_{S_c}-0.3e_{s_u}}
\end{aligned}$$

$$\gamma_{Ss}^{\eta} = \eta \text{ quantile of } e^{\varepsilon_{S_s}-e_{\sigma'_{vs}}} \qquad \gamma_{Sg}^{\eta} = \eta \text{ quantile of } e^{\varepsilon_{S_g}-e_{\sigma'_{vg}}} \qquad \gamma_{Sr}^{\eta} = \eta \text{ quantile of } e^{\varepsilon_{S_r}-0.414e_{q_u}}$$

$$\begin{aligned}
\gamma_{LD}^{\eta} &= \left[(1-\eta) \text{ quantile of } L_D(Z,\theta)\right] / L_D^* = (1-\eta) \text{ quantile of } \left[L_D(Z,\theta)/8000\right] \\
\gamma_{LL}^{\eta} &= (1-\eta) \text{ quantile of } \left[L_L(Z,\theta)/4000\right]
\end{aligned} \tag{46}$$

Figure 6 shows the relations between η and the MRFD design factors.

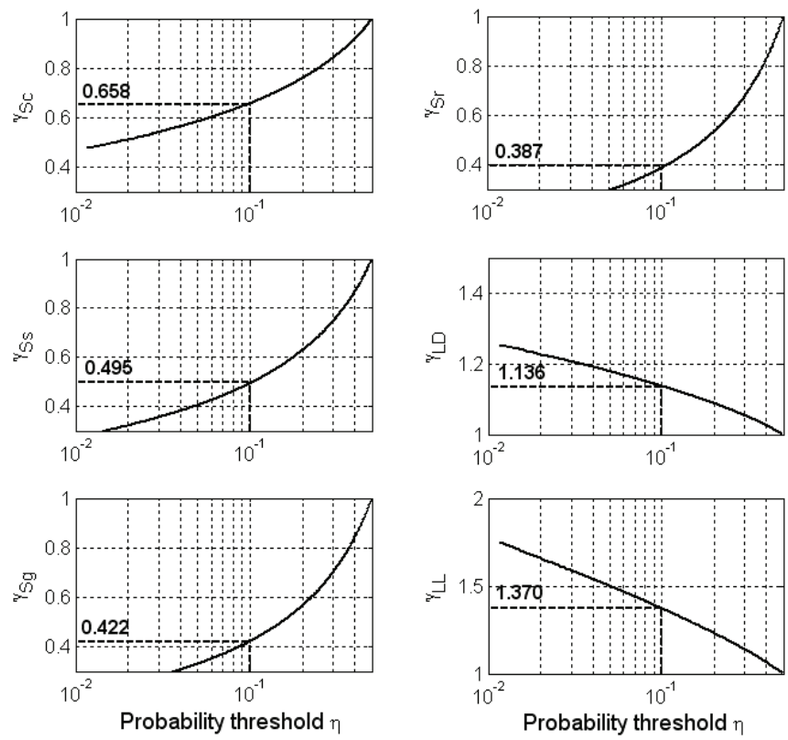

Fig. 6. Relations between η and the MRFD design factors.

Step 2 – find the relation between (η,P_F^*) by solving Eq. (43). For instance, when η = 0.1, the corresponding design factors can be readily from Figure 6 to be γ_{Sc} = 0.658, γ_{Ss} = 0.495, γ_{Sg} = 0.422, γ_{Sr} = 0.387, γ_{LD} = 1.136, and γ_{LL} = 1.370. Therefore,

$$\frac{c(\theta)}{SR(Z,\theta)}=\frac{\left[\gamma_{Sc}^{\eta}S_c^*+\gamma_{Ss}^{\eta}S_s^*+\gamma_{Sg}^{\eta}S_g^*+\gamma_{Sr}^{\eta}S_r^*\right]\times\left[L_D(Z,\theta)+L_L(Z,\theta)\right]}{\left[S_c(Z,\theta)+S_s(Z,\theta)+S_g(Z,\theta)+S_r(Z,\theta)\right]\times\left[\gamma_{LD}^{\eta}L_D^*+\gamma_{LL}^{\eta}L_L^*\right]}$$

$$=\frac{\begin{bmatrix}0.658e^{2.7+0.3\ln s_{u,m}}t_c+0.495e^{1.0802-0.6588\ln(d_s)+\ln(\sigma'_{vs,m})}t_s\\+0.422e^{2.1792-0.7528\ln(d_g)+\ln(\sigma'_{vg,m})}\min\left(L-t_c-t_s,t_g\right)\\+0.387e^{3.0253+0.414\ln q_{u,m}}\min\left(L-t_c-t_s-t_g,0\right)\end{bmatrix}\cdot\left[L_D(Z,\theta)+L_L(Z,\theta)\right]}{\begin{bmatrix}e^{2.7+0.3\left[\ln s_{u,m}-e_{s_u}\right]+\varepsilon_{S_c}}t_c+e^{1.0802-0.6588\ln(d_s)+\left[\ln(\sigma'_{vs,m})-e_{\sigma'_{vs}}\right]+\varepsilon_{S_s}}t_s\\+e^{2.1792-0.7528\ln(d_g)+\left[\ln(\sigma'_{vg,m})-e_{\sigma'_{vg}}\right]+\varepsilon_{S_g}}\min\left(L-t_c-t_s,t_g\right)\\+e^{3.0253+0.414\left[\ln q_{u,m}-e_{q_u}\right]+\varepsilon_{S_r}}\min\left(L-t_c-t_s-t_g,0\right)\end{bmatrix}\cdot\left[1.136\times8000+1.370\times4000\right]} \tag{47}$$

According to Eq. (43), the corresponding P_F^* is exactly P(c(θ)/SR(Z,θ)>1). Monte Carlo simulation can be taken to simulate many c(θ)/SR(Z,θ) samples estimate, and the estimate of

the probability of exceeding 1 is exactly the P_F^* value corresponding to $\eta = 0.1$. Note that for the MCS, the design parameters B and L should be taken to be random and uniformly distributed over the allowable design region 0.8 m ≤ B ≤ 1.5 m and 49.6 m ≤ L ≤ 74.7 m. The entire (η, P_F^*) relation can be obtained by changing the probability threshold η and conduct the same MCS. Figure 7 shows the resulting relation between (η, P_F^*). Since the target failure probability P_F^* is 0.001, the required probability threshold should be 0.071 (see Figure 7). By inverting Figure 6 with $\eta = 0.071$, it can be found that $\gamma_{Sc} = 0.618$, $\gamma_{Ss} = 0.446$, $\gamma_{Sg} = 0.372$, $\gamma_{Sr} = 0.336$, $\gamma_{LD} = 1.158$, and $\gamma_{LL} = 1.436$. These are the MRFD factors that should be taken for a RBD with $P_F^* = 0.001$, and the resulting algebraic design constraint is

$$c(\theta) = \frac{0.618 \times S_c^* + 0.446 \times S_s^* + 0.372 \times S_g^* + 0.336 \times S_r^*}{1.158 \times 8000 + 1.436 \times 4000} \geq 1 \tag{48}$$

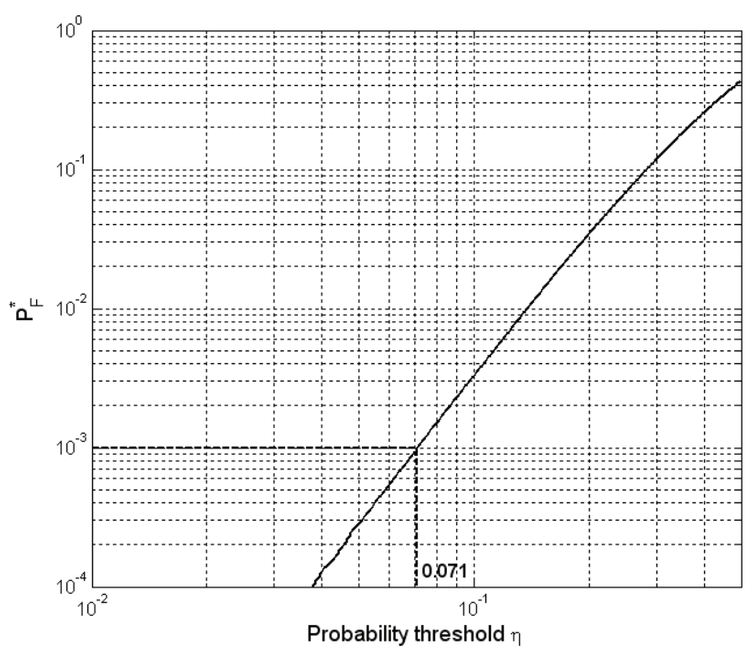

Fig. 7. Relation between (η, P_F^*)

To examine the robustness of the resulting MRFD design factors for $P_F^* = 0.001$, the following approach is taken. In the allowable design region 0.8 m ≤ B ≤ 1.5 m and 49.6 m ≤ L ≤ 74.7 m, each of the coordinate axes is discretized into discrete points, creating grid points. MCS with a very large sample size is then conducted at each grid point, giving each point an independent estimate of the failure probability $P(SR(Z,\theta)<1 \mid \theta)$. The dividing boundary for $P(SR(Z,\theta)<1 \mid \theta)$ less and greater than 0.001 is plotted shown in Figure 8. Therefore, the region above the boundary is exactly the allowable reliability design set $\Sigma_R = \{\theta: P(SR(Z,\theta)<1 \mid \theta) \leq P_F^*\}$. On the other hand, the allowable design region for the algebraic constraint, i.e., $\Sigma_S = \{\theta: c(\theta) \geq 1\}$, can be also found and is marked with label 'o'. For the unsatisfactory region with $c(\theta) < 1$, it is marked with label 'x'. Such comparisons are made in Figure 8, not only for the MRFD design format but also for the other three aforementioned design formats, although the detailed steps of those three design formats are not presented.

It can be seen from Figure 8 that the two sets match one another reasonably well for all four design methods.

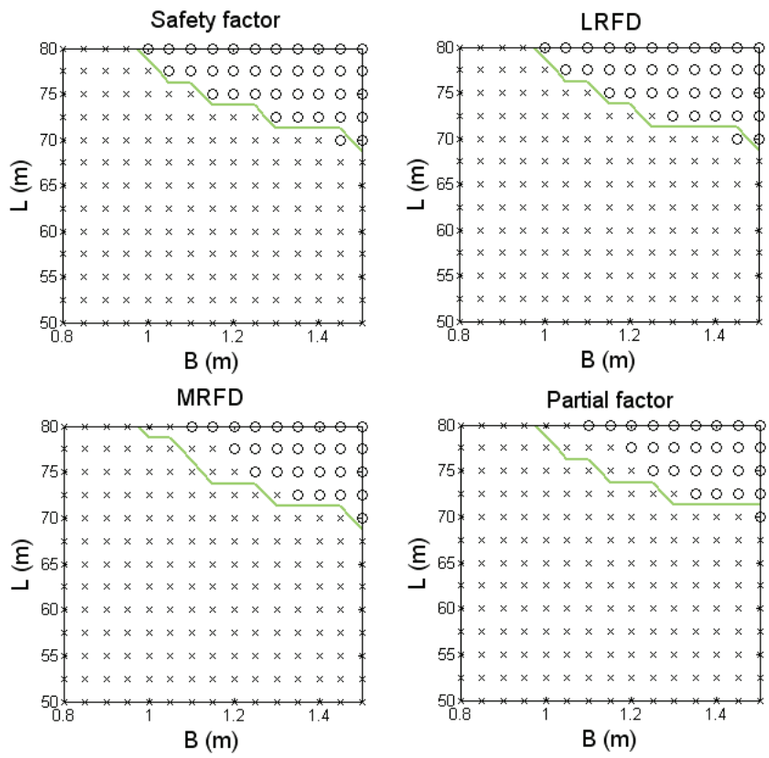

Fig. 8. Comparison between algebraic design constraints and rigorous RBD

4.5 Iterative design process

Given the required MRFD factors γ_{Sc} = 0.618, γ_{Ss} = 0.446, γ_{Sg} = 0.372, γ_{Sr} = 0.336, γ_{LD} = 1.158, and γ_{LL} = 1.436, the goal now is to design the diameter B and length L of the drilled shaft so that $c(\theta) \geq 1$. The resulting design should also satisfy failure probability less than 0.001. The design process is iterative. Let us start with a design with diameter = 1.2 m and depth = 70 m. Based on this design dimension and also the information in Table 1, the nominal resistance can be computed from Eq. (45): S_c^* = 8448 kN, S_s^* = 2345 kN, S_g^* = 14808 kN and S_r^* = 389 kN. Note that when calculating S_c^*, S_s^*, S_g^*, S_r^*, the characteristic values of the geotechnical parameters must be fixed at the measured values (or average values) of those parameters. Now compute

$$\begin{aligned} c(\theta) &= \frac{\gamma_{Sc}^{\eta} S_c^* + \gamma_{Ss}^{\eta} S_s^* + \gamma_{Sg}^{\eta} S_g^* + \gamma_{Sr}^{\eta} S_r^*}{\gamma_{LD}^{\eta} \cdot L_D^* + \gamma_{LL}^{\eta} \cdot L_L^*} \\ &= \frac{0.618 \cdot 8448 + 0.446 \cdot 2345 + 0.372 \cdot 14808 + 0.336 \cdot 389}{1.158 \cdot 8000 + 1.436 \cdot 4000} = 0.793 < 1 \end{aligned} \tag{49}$$

It is therefore concluded that the design is not satisfactory for target failure probability less than 0.001. A design with greater length or larger diameter is needed, and iterations should be taken until $c(\theta)$ is greater than 1.

5. Conclusion

Monte Carlo based reliability analysis methods and reliability-based design methods are introduced in this chapter. A realistic geotechnical design example is developed and is used to demonstrate the uses of all methods. The main benefits for the Monte Carlo based methods include (a) their implementations do not require the knowledge of optimization skills, as required by the first-order reliability methods; and (b) they are mostly general and robust to the dimension of random variables and problem complexity, hence ideal for geotechnical problems. One possible drawback is that these Monte Carlo based methods are more time consuming, but this issue has been alleviated greatly due to recent development of powerful personal computers. As a result, these methods are believed to be ideal for practical implementations.

6. Appendix derivations for the equivalence principle

The safety ratio SR(Z,θ) is a positive-valued random variable, taking values strictly larger than 0, i.e., SR(Z,θ) > 0, and c(θ) is a positive-valued deterministic function of θ.

$$\begin{aligned} & P\left(SR(Z,\theta)<1\,|\,\theta\right) \\ & = P\left(0<SR(Z,\theta)<1\,|\,\theta\right) \\ & = P\left(1/SR(Z,\theta)>1\,|\,\theta\right) && \text{because } SR>0 \\ & = P\left(c(\theta)/SR(Z,\theta)>c(\theta)\,|\,\theta\right) && \text{because } c>0 \end{aligned} \tag{50}$$

Observe that P(c(θ)/SR(Z,θ) > x | θ) is a non-increasing (i.e. equal or decreasing) function with x (a constant), since P(c(θ)/SR(Z,θ) > x | θ) = P(SR(Z,θ) < c(θ)/x | θ) is cumulative distribution function of SR(Z,θ) which is a non-decreasing (i.e. equal or increasing) function with c(θ)/x by definition.
Hence, if c(θ) ≥ 1,

$$P\left(c(\theta)/SR(Z,\theta)>c(\theta)\,|\,\theta\right)\le P\left(c(\theta)/SR(Z,\theta)>1\,|\,\theta\right) \tag{51}$$

If we further let

$$P\left(c(\theta)/SR(Z,\theta)>1\,|\,\theta\right)=P_F^* \tag{52}$$

Then,

$$P\left(SR(Z,\theta)<1\,|\,\theta\right)=P\left(c(\theta)/SR(Z,\theta)>c(\theta)\,|\,\theta\right)\le P\left(c(\theta)/SR(Z,\theta)>1\,|\,\theta\right)=P_F^* \tag{53}$$

Note that the hypothesis is required for practicality. The above constitutes a proper proof of equivalence if we allow η to be a function of θ. In summary, the practical usefulness of the equivalence between the statement {θ: c(θ) ≥ 1} and {θ: P(SR(Z,θ)<1 | θ) ≤ P_F^*} is predicated on the possibility of finding a η which is *not* a function of θ for any prescribed target failure probability, P_F^*.

7. References

Ang, A.H.-S. & Tang, W.H. (1984). *Probability Concepts in Engineering Planning and Design, Vol. I: Basic Principles.* John Wiley & Sons.

Au, S. K., Papadimitriou, C. & BECK, J.L. (1999). Reliability of uncertain dynamical systems with multiple design points. *Structural Safety*, 21, 113-133.

Au, S.K. & Beck, J.L. (2001). Estimation of small failure probability in high dimensions by subset simulation. *Probabilistic Engineering Mechanics*, 16, 263-277.

Au, S.K. & Beck, J.L. (2003). Importance sampling in high dimensions. *Structural Safety*, 25(2), 139-163.

Ching, J. & Phoon, K.K. (2010), Quantile framework for simplified geotechnical reliability-based design. To appear in *Proceedings of 2011 Georisk Conference*.

Ching, J., Beck, J.L. & Au, S.K. (2005). Hybrid subset simulation method for reliability estimation of dynamic systems subject to stochastic excitations. *Probabilistic Engineering Mechanics*, 20(3), 199-214.

Der Kiureghian, A. (2000). The geometry of random vibrations and solutions by FORM and SORM. *Probabilistic Engineering Mechanics*, 15, 81-90.

Der Kiureghian, A. & Dakessian, T. (1998). Multiple design points in first and second-order reliability. *Structural Safety*, 20, 37-49.

Ghanem, R. & Spanos, P. (1991). A spectral stochastic finite element formulation for reliability analysis. *Journal of Engineering Mechanics, ASCE*, 117(10), 2351-2372.

Hasofer, A.M. & Lind, N.C. (1974). Exact and invariant second-moment code format. *Journal of Engineering Mechanics*, ASCE, 100(1), 111-121.

Homenbichler, M. & Rackwitz, R. (1988). Improvement of second-order reliability estimates by importance sampling. *Journal of Engineering Mechanics*, ASCE, 114(12), 2195-2198.

Liu, P.L. & Der Kiureghian, A. (1991). Optimization algorithms for structural reliability. *Structural Safety*, 9(3), 161-177.

Melchers, R.E. (1989). Importance sampling in structural systems. *Structural Safety*, 6, pp.3-10.

Applications of Monte Carlo Method in Science and Engineering
Edited by Prof. Shaul Mordechai

ISBN 978-953-307-691-1
Hard cover, 950 pages
Publisher InTech
Published online 28, February, 2011
Published in print edition February, 2011

In this book, Applications of Monte Carlo Method in Science and Engineering, we further expose the broad range of applications of Monte Carlo simulation in the fields of Quantum Physics, Statistical Physics, Reliability, Medical Physics, Polycrystalline Materials, Ising Model, Chemistry, Agriculture, Food Processing, X-ray Imaging, Electron Dynamics in Doped Semiconductors, Metallurgy, Remote Sensing and much more diverse topics. The book chapters included in this volume clearly reflect the current scientific importance of Monte Carlo techniques in various fields of research.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Jianye Ching (2011). Practical Monte Carlo Based Reliability Analysis and Design Methods for Geotechnical Problems, Applications of Monte Carlo Method in Science and Engineering, Prof. Shaul Mordechai (Ed.), ISBN: 978-953-307-691-1, InTech, Available from: http://www.intechopen.com/books/applications-of-monte-carlo-method-in-science-and-engineering/practical-monte-carlo-based-reliability-analysis-and-design-methods-for-geotechnical-problems

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License, which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.