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## Cooperative Logistics Games

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### 1. Introduction

Roughly speaking, Game Theory deals with analysing conflict and cooperation situations in which two or more rational and intelligent agents are involved. There are many real and theoretical situations which can be examined from the point of view of Game Theory. Therefore it is not difficult to find in the literature a rich variety of applications of Game Theory to many and very diverse fields of knowledge. In particular, Game Theory plays a significant role in Economics, but we can also find applications to Computer Science and Engineering.

Game Theory can be roughly divided into two main areas: cooperative and non cooperative games. The basic key for distinguishing between these two areas is whether it is possible or not to reach binding agreements. When binding agreements are possible, we are then faced with a cooperative situation. Thus, in a cooperative environment the concept of coalition plays an important role and very often the main goal is to achieve the cooperation of all agents. In this chapter we will assume that binding agreements among the agents are possible and therefore we will use the cooperative approach for analysing some logistics problems.

On the other hand, there are a number of theoretical and conceptual connections between Game Theory and Operations Research (OR). For example, we should mention the connection between the duality in mathematical programming and the minimax theorems for zero-sum games (see Raghavan, 1994); the linear complementary theory and the bi-matrix games (see Lemke, 1965), or the optimal control theory and the differential games (see Friedman, 1994) among others. Furthermore we can find applications of OR to Game Theory, for example the characterization of balanced games using the duality concept (Bondareva, 1963 and Shapley, 1967). Likewise, Game Theory contributes to completing the analysis of OR problems when there is more than one agent involved in the corresponding situation. Thus, after optimising a particular system by means of OR techniques, in which there are two or more agents involved, who have to collaborate in order to be able to achieve that optimal result, saying something about how to distribute the extra benefits or the costs saved by cooperation among those agents seems reasonable and necessary. Hence cooperative games can play a role in the complete analysis of the situation.

In the literature, not only we can find many OR problems studied from the point of view of cooperative games in the sense mentioned previously, but also OR problems analysed from

a strategic or non-cooperative approach. However, in this chapter we are more interested in the cooperative approach. Some of the first OR situations studied using cooperative games are assignment problems (Shapley & Shubik, 1971), linear production problems (Owen, 1975), network flow problems (Kalai & Zemel, 1982) and minimum cost spanning tree problems (Claus & Kleitman, 1973 and Bird, 1976), obtaining the so-called assignment games, linear production games and so on. The games obtained from OR problems are usually called OR-games (see Borm et al., 2001 for a survey on this topic).

In general, the methodology to analyse an OR problem from a cooperative approach consists of associating a coalitional game to each problem or characteristic function form game summarising the gains or savings from cooperation for each possible coalition of the agents involved and, thereby, analysing different topics of Game Theory such as solution concepts, stability, etc. Thus we can try to answer the question posed before, namely, 'How to distribute the extra benefits or the costs saved by achieving cooperation among the different agents involved'.

Logistics include the analysis and management of many different situations which can be formulated or modelled as OR problems. Thus problems related to transportation, inventory, supply chain, distribution, location, routing or storage among others, arise frequently in logistics. One can also consider that all of these problems may have more than one agent involved, so a game theoretical approach could be used to tackle them either from a cooperative point of view or from a non cooperative point of view. In the literature we can find both approaches for the different logistics problems but we will concentrate our attention on the cooperative approach.

In this chapter we will only analyse two logistics problems from a cooperative point of view: transportation situations –and some related problems– and supply chain situations. The two problems selected are representative of a particular problem in logistics, such as the transportation of goods from stores or production sources to points of sale or distribution and a general problem, such as the supply chain which embraces many (or all) logistics tasks. Therefore we have selected one particular problem and a more general problem. In this sense it is possible to consider logistics as being a part of supply chain management but we have considered the supply chain inside logistics in order to be able to analyse separately different interesting optimisation problems under the same umbrella. On the other hand, we are aware that these two problems do not cover all possible logistics situations but we believe that the analysis of these problems together with the references provided throughout the chapter can provide a good starting point for the reader interested in this topic.

Finally, since we will use the cooperative approach to analyse the different problems and hence are interested in cooperation between the agents, then we will study the concept of coalitional stability represented by the core of the game. To this end, we will analyse the non-emptiness of the core of the corresponding game and therefore the existence of coalitional stable distributions. Likewise, we will explore other possible solution concepts and their relationship to the core of the game.

The rest of the chapter is organised as follows. In Section 2 we provide the basic definitions, concepts and solutions of cooperative games. We also describe the methodology for defining a cooperative OR game and introduce logistics games. Section 3 analyses the cooperative approach for transportation situations and some related problems which can arise in logistics situations. In Section 4 we review the literature for the cooperative approach for

supply chain situations and explore the possibility to analyse from a cooperative standpoint supply chain situations without storage through two particular examples. Finally, in Section 5 we briefly revise the literature for other logistics games.

## 2. Preliminaries

In this section we formally introduce some basic definitions, concepts and solutions for cooperative games in order to provide the reader with all the necessary background to follow this chapter. Likewise, we present what we mean for Operations Research Games and the definition of logistics games.

### 2.1 Basic notions on cooperative games

First, a *cooperative game in characteristic function form* is a pair  $(N, v)$  where  $N$  is a finite set of agents called players and  $v$  is a function that associates to each set  $S \subset N$  a real value  $v(S)$  satisfying  $v(\emptyset)=0$ . This value  $v(S)$  represents the joint gain that the agents in  $S$  can guarantee by themselves if they cooperate independently of what the agents in  $N \setminus S$  could do. Therefore, in some sense,  $v(S)$  measures the worth of coalition  $S$ . On the other hand, when the characteristic function represents costs instead of gains or benefits then we will denote it by  $c$  and we refer to *cost games*. Of course, it is possible to transform a cost game  $(N, c)$  in a benefit game through the so-called *savings game*. The definition of a savings game  $(N, v^c)$  associated with a cost game  $(N, c)$  is the following:

$$v^c(S) = \sum_{i \in S} c(i) - c(S). \quad (1)$$

Therefore the savings game is simply the saved costs from cooperation with respect to all the individual costs. Thus, the savings game represents the gains of cooperation as opposed to acting separately.

We will denote by  $G^N$  the set of all (benefit or profit) games with set of players  $N$  and by  $CG^N$  the set of all cost games with set of players  $N$ . Furthermore, we will denote by  $G$  the set of all (benefit or profit) games and by  $CG$  the set of all cost games.

There are some properties of the characteristics function which, at first glance, if a game satisfies them, then it seems that cooperation is profitable for the agents and hence the possibility of cooperation exists. However, a more careful analysis is necessary as we will see later.

For profit or benefit games the properties are the following:

- *Monotonicity*: if  $v(S) \leq v(T)$  for all  $S \subset T \subset N$ .
- *Superadditivity*: if  $v(S \cup T) \geq v(S) + v(T)$  for all  $S, T \subset N$  such that  $S \cap T = \emptyset$ .
- *Convexity*: if  $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$  for all  $S, T \subset N$ .

For cost games their counterparts can be written as:

- *Monotonicity*: if  $c(T) \leq c(S)$  for all  $S \subset T \subset N$ .
- *Subadditivity*: if  $c(S \cup T) \leq c(S) + c(T)$  for all  $S, T \subset N$  such that  $S \cap T = \emptyset$ .
- *Concavity*: if  $c(S \cup T) + c(S \cap T) \leq c(S) + c(T)$  for all  $S, T \subset N$ .

Given a game  $(N, v)$  (resp. cost game  $(N, c)$ ) a *distribution or allocation* for it is a vector  $z \in \mathbb{R}^N$  such that  $\sum_{i \in N} z_i \leq v(N)$  (resp.  $\sum_{i \in N} z_i \geq c(N)$ ). We will denote by  $z(S) = \sum_{i \in S} z_i$ . A distribution  $z$  is called *efficient* if  $z(N) = v(N)$  (resp.  $z(N) = c(N)$ ).

A *solution* for  $G$  (resp.  $CG$ ) is a map  $\sigma:G \rightarrow \mathfrak{R}^N$  (resp.  $\sigma:CG \rightarrow \mathfrak{R}^N$ ) such that  $\sigma(N, v) \subset \mathfrak{R}^N$  for all  $(N, v) \in G$  (resp.  $\sigma(N, c) \subset \mathfrak{R}^N$  for all  $(N, c) \in CG$ ) and  $z(N)=v(N)$  (resp.  $z(N)=c(N)$ ) for all  $z \in \sigma(N, v)$ . If  $\sigma$  is always a single point then it is called *value*, otherwise it is called a set-valued solution or simply a solution. A solution for a game is a set of efficient distributions of the total gain or cost. One of the most outstanding solutions is the *core*. The core of a game is the set of all coalitional stable distributions and, therefore, any coalition obtains at least what the members of it can achieve by themselves. In formulas for benefit/profit games and cost games respectively:

$$\text{Core}(N, v) = \{z \in \mathfrak{R}^N : z(S) \geq v(S) \text{ for all } S \subset N \text{ and } z(N) = v(N)\}. \quad (2)$$

$$\text{Core}(N, c) = \{z \in \mathfrak{R}^N : z(S) \leq c(S) \text{ for all } S \subset N \text{ and } z(N) = c(N)\}. \quad (3)$$

The distributions in the core of a game are interesting because there is no incentive for any coalition to reject them. However, the core of a game can be empty. The games with non-empty core are called *balanced*. (Shapley, 1971) proved that all convex games (resp. concave for the case of cost games) have a non-empty core and hence they are balanced.

On the other hand, another interesting set of distributions is the *imputation set*. It is defined as the set of all efficient and individually stable (or rational) distributions. In formulas for benefit/profit games and cost games respectively:

$$I(N, v) = \{z \in \mathfrak{R}^N : z_i \geq v(i) \text{ for all } i \in N \text{ and } z(N) = v(N)\}. \quad (4)$$

$$I(N, c) = \{z \in \mathfrak{R}^N : z_i \leq c(i) \text{ for all } i \in N \text{ and } z(N) = c(N)\}. \quad (5)$$

Given a game  $(N, v)$  the *marginal contribution* of player  $i$  to coalition  $S$  ( $i \notin S$ ) is given by  $v(S \cup i) - v(S)$  (resp.  $c(S \cup i) - c(S)$ ). Based on this concept another outstanding solution for cooperative games is defined: the *Shapley value* (Shapley, 1953). For each player the Shapley value is the average of all her possible marginal contributions. The mathematical expression of the Shapley value is the following:

$$Sh_i(N, v) = \sum_{S \subset N, i \notin S} \gamma_n(S) [v(S \cup i) - v(S)], \forall i \in N \quad (6)$$

where  $\gamma_n(S) = \frac{s!(n-s-1)!}{n!}$  and  $s = \text{card}(S)$ .

The Shapley always exists but does not belong to the core in general. However, (Shapley, 1971) proved that if the game is convex (resp. concave for cost games) then the Shapley value is always in the core of the game.

(Schmeidler, 1969) introduced a value, called *nucleolus*, which always belongs to the core of the game when it is non-empty. The definition of the nucleolus is based on the concept of *excess (or complaint) of a coalition with regard to a distribution*. Given a game  $(N, v)$  (resp.  $(N, c)$ ), a coalition  $S \subset N$  and a distribution  $z$ , the excess of coalition  $S$  with regard to distribution  $z$  is given by  $e(S; z) = v(S) - z(S)$  (resp.  $e(S; z) = z(S) - c(S)$ ). Likewise, we define  $\theta(z)$  as the vector of all excesses with regard to  $z$  written in decreasing order. The nucleolus of a game  $(N, v)$  (analogously for a cost game  $(N, c)$ ) is defined as

$$nu(N, v) = \{z \in I(N, v) : \theta(z) \leq_L \theta(x) \text{ for all } x \in I(N, v)\}, \quad (7)$$

where  $\leq_L$  is the lexicographic order. Therefore, the nucleolus is the distribution that minimises the maximal excess or complaint of all coalitions.

There are a number of different solutions for cooperative games in characteristic function form. For this reason it is necessary to know which solutions are more suitable for a particular situation. One way to understand the solutions better is through the properties they satisfy. The main objective is to know which “reasonable” properties characterise each solution. Thus, depending on which properties are meaningful or important in a particular situation, we would be able to find out which solutions fit better too. Therefore, we can find many papers in the literature characterising solutions for cooperative games using different sets of properties.

## 2.2 Cooperative Operations Research Games (ORGs)

Consider a system where there are one or more agents interested in optimising it. One way to deal with this situation is to have recourse to Operations Research and we are then faced with an operations research problem. The simpler situation is when there is only one agent or decision-maker involved in the problem and, therefore, there is no conflict of interests. In that case the analysis of the system is completed on the procurement of one optimal solution for it using the appropriate optimisation techniques. However, it is not difficult to find that, on many occasions, there would be more than one agent or decision-maker involved in the system and, consequently, some kind of conflict of interests could arise. In that case, each agent could own or control one or more parts of the system and if they wanted to optimise the system then they should cooperate but, perhaps, they should agree on how to distribute the profits/benefits or saved costs among themselves. Therefore, the analysis of the systems does not end with the procurement of one optimal solution but it is necessary to go a step further in order to convince the agents involved to cooperate, most likely, via a good distribution of the profits or saved costs. One way to tackle this last step in the analysis is using cooperative games.

Given an operations research problem  $A$  in which there is a finite set  $N$  of agents involved, we define an associated cooperative game in characteristic function form  $(N, v^A)$  in the following way:

$$\begin{aligned} v^A(\emptyset) &= 0, \\ v^A(N) &= \text{Optval}(A) \text{ and} \\ v^A(S) &= \text{Optval}(A_S) \text{ for all } S \subset N, \end{aligned} \quad (8)$$

where  $\text{Optval}(A)$  is the optimal value for problem  $A$  and  $\text{Optval}(A_S)$  is the optimal value for problem  $A_S$ , where  $A_S$  is the problem obtained using only the parts of problem  $A$  owned or controlled by the agents in coalition  $S$ . In the case that problem  $A$  is a cost problem we can analogously define the cost game  $(N, c^A)$ . These games are called (cooperative) *operations research games*. Furthermore, if the operations research problems are related to logistics situations then we will call them *cooperative logistics games*.

Once we have defined a cooperative game associated with an operations research problem, then we could obtain different answers to the question of how to distribute the profits/benefits or saved costs among the agents involved using the solutions defined for cooperative games, such as the core, the Shapley value, the nucleolus, etc. Note that if we only use the characteristic function of the game then we may lose some of the essence of the

problem. However, it would also be possible to think of the primal and dual optimal solutions of the operations research problem to obtain distributions of the profits/saved costs among the agents involved. Thus, in the latter approach, we would be considering, in some manner, the particular features of the operations research problem. Of course, the choice of one approach or another will depend on the particular situation.

Two examples of solutions based on the primal optimal solutions of the corresponding operations research problems are the Bird solution for minimum cost spanning tree games (Bird, 1976) which is based on the application of the Prim algorithm (Prim, 1957) and the pairwise solutions for transportation games (Sanchez-Soriano, 2003 and 2006). The first is a solution based on an algorithm while the second are solutions based directly on the optimal solutions of the problem. Therefore, we have two different examples of how to use the Operations Research techniques to obtain the distribution of the total profits/saved costs among the agents taking part in the problem. In both cases the relationship between the solution and the core of the game is studied.

Another possibility is to deal with the optimal solutions of the dual problem. Two examples of this approach are (Shapley & Shubik, 1971) for assignment problems and (Owen, 1975) for linear production problems. In the first paper, the authors proved that the core of the game and the set of dual optimal solutions coincide. In the second paper, the inclusion of the set of distributions based on the dual optimal solutions in the core of the game is demonstrated. The set of distributions based on the dual optimal solutions is called the *Owen set* (van Gellekom et al., 2000).

### 3. Transportation, distribution and warehouse sharing games

In this section we will study some transportation problems from the point of view of cooperative games. We will start with the simplest transportation situation with only two types of agents (suppliers and demanders) which we call two-sided transportation problem. A problem of this kind describes three possible logistics situations of transportation of goods: producers-retailers, producers-wholesalers or wholesalers-retailers. In each case, the mathematical treatment of these is essentially the same. Secondly, we will analyse transportation situations with three types of agents (suppliers, intermediates and demanders) which we call three-sided transportation problems. A situation of this kind corresponds to producers-wholesalers-retailers distribution problems. Finally, we will study warehouse sharing problems in which the agents involved in the situation must share the warehouses in order to optimise their transportation profits/costs.

#### 3.1 Two-sided transportation games

Basically, a two-sided transportation problem consists of two sets of agents, called producers and retailers, which produce and demand goods. Each producer produces a quantity of goods and each retailer demands a certain amount of goods. The transport of the goods from the producers to the retailers is costly (profitable) and, therefore, the main objective is to transport the goods from the producers to the retailers at minimum cost (at maximum profit). The way to achieve this objective is by means of cooperation, otherwise if each agent would make decisions on their own, then the final result of the transportation would be unpredictable and, perhaps, far from the optimal situation. Therefore, if cooperation is profitable then this should be promoted through a good distribution of the extra profits or saved costs.

Let  $P$  and  $R$  be the sets of producers and retailers respectively. We denote by  $p_i$  the production of goods of producer  $i \in P$  and by  $d_j$  the demand of goods of retailer  $j \in R$ . The unitary cost (resp. benefit) of transportation from producer  $i$  to retailer  $j$  is denoted by  $c_{ij}$  (resp.  $b_{ij}$ ). The mathematical model of this problem can be described by:

$$\begin{aligned} \min \quad & \sum_{i \in P} \sum_{j \in R} c_{ij} x_{ij} \\ \text{s.t. :} \quad & \sum_{j \in R} x_{ij} \leq p_i, i \in P \\ & \sum_{i \in P} x_{ij} \geq d_j, j \in R \\ & x_{ij} \geq 0, i \in P, j \in R \end{aligned} \quad (9)$$

where  $x_{ij}$  is the number of units transported from producer  $i$  to retailer  $j$ .

Problem (9) has feasible solutions if it satisfies that  $\sum_{i \in P} p_i \geq \sum_{j \in R} d_j$ . However, if we consider

that each transported unit lead up to a benefit  $b$  (large enough to compensate any unitary cost) then we can consider a maximisation problem with coefficients  $b_{ij} = b - c_{ij}$  and relax the second block of constraints by changing the direction of the inequalities. This new problem has always got feasible solutions and that drawback is avoided. Therefore, from now on, we will consider transportation problems with benefits instead of costs. Consequently, the corresponding mathematical program is given by

$$\begin{aligned} \max \quad & \sum_{i \in P} \sum_{j \in R} b_{ij} x_{ij} \\ \text{s.t. :} \quad & \sum_{j \in R} x_{ij} \leq p_i, i \in P \\ & \sum_{i \in P} x_{ij} \leq d_j, j \in R \\ & x_{ij} \geq 0, i \in P, j \in R. \end{aligned} \quad (10)$$

Now, we can define a cooperative game in characteristic function form associated with each (benefit) transportation problem  $T$ . The set of players  $N = P \cup R$  and the characteristic function  $v^T$  is defined following the general formulas given in (8). The game  $(N, v^T)$  is called *transportation game*. Transportation games are superadditive but not convex in general. Furthermore, the core of these games is always non-empty. On the other hand, if  $(u; w)$  is an optimal solution for the dual problem of (10), then  $((p_i u_i)_{i \in P}; (d_j w_j)_{j \in R}) \in \text{Core}(N, v^T)$ . Therefore, the *Owen set*  $(N, v^T) = \{((p_i u_i)_{i \in P}; (d_j w_j)_{j \in R}) \in \mathfrak{R}^{P \cup R}; (u; w) \text{ is a dual optimal solution}\}$  is contained in the core of the game. However, the core and the Owen set of transportation games do not coincide in general (see Sanchez-Soriano et al., 2001). In (Thompson, 1980) the extreme points of the Owen set that the author called "core" are studied.

In (Sanchez-Soriano, 2003 and 2006) the *pairwise solutions* for transportation games are introduced. These solutions are based directly on the optimal solutions of the corresponding transportation problem. Since transportation problems can have more than one optimal solution, the pairwise solutions are set-valued (but discrete). However, on many occasions, transportation problems have only one optimal solution and, hence, we could consider that pairwise solutions are "essentially" values. The philosophy behind the pairwise solutions is



simply that the benefit obtained by each pair producer-retailer in an optimal solution is distributed between them in some way. The proportion of benefit achieved for a player in a pair producer-retailer will depend on the bargaining abilities of both or on their relative weight (power) in the whole transportation system. When we assume that nothing is known about the relative weights of the agents and, therefore, we could consider that they all have the same weight, then we obtain the *pairwise egalitarian solution*. Given a weight vector  $\pi$ , such that  $\pi_k > 0$  for all  $k \in N$ , and an optimal solution  $x^*$  for the corresponding problem (10), the pairwise solution associated with  $\pi$  and  $x^*$  is defined as follows:

$$\begin{aligned} ps_i(\pi, x^*) &= \sum_{j \in R} \frac{\pi_i}{\pi_i + \pi_j} b_{ij} x_{ij}^*, i \in P \\ ps_j(\pi, x^*) &= \sum_{i \in P} \frac{\pi_j}{\pi_i + \pi_j} b_{ij} x_{ij}^*, j \in R. \end{aligned} \quad (11)$$

The *pairwise solution with weight vector*  $\pi$  for the game  $(N, v^T)$  is defined as

$$PS^\pi(N, v^T) = \{ps(\pi, x^*) \in \mathfrak{R}^{P \cup R} : x^* \in Opt(T)\}, \quad (12)$$

where  $Opt(T)$  is the set of all optimal solutions for the corresponding transportation problem  $T$ .

On the other hand, we could use a more general concept as the weight systems (Kalai & Samet, 1987) instead of a simple weight vector. A weight system on a set  $N$  is a pair  $(\Sigma, \pi)$  where  $\Sigma$  is a partition of  $N$ ,  $(N_1, N_2, \dots, N_q)$ , and  $\pi$  is a weight vector, whose coordinates are ordered in the same order as the partition. Such that the weight of agents in  $N_h$  is zero with respect to the agents in  $N_k$  if  $h < k$ . Inside of each  $N_h$  each agent has a positive weight. In this situation we can define the *pairwise solution with weight system*  $(\Sigma, \pi)$  for the game  $(N, v^T)$ ,  $PS^{(\Sigma, \pi)}(N, v^T)$ , analogously to (11) and (12). The pairwise solutions do not belong to the core of the game in general, but in (Sanchez-Soriano, 2006) it is proved that

$$Core(N, v^T) \subset \bigcup_{(\Sigma, \pi)} PS^{(\Sigma, \pi)}(N, v^T). \quad (13)$$

Therefore, each core allocation can be seen as a pairwise solution for particular weight systems but there are, in general, pairwise solutions which do not belong to the core of the corresponding transportation game.

Let us consider a transportation situation  $T$  with two producers (called  $A$  and  $B$ ) and three retailers (called 1, 2, and 3). The productions of  $A$  and  $B$  are 12 and 15 units respectively and the demand of each retailer is 10 units. The unitary costs of transportation are  $c_{A1}=3$ ,  $c_{A2}=5$ ,  $c_{A3}=6$ ,  $c_{B1}=5$ ,  $c_{B2}=4$  and  $c_{B3}=3$ . And the unitary benefit obtained by each good is 9. Solving the corresponding transportation problem (10), we obtain that the only optimal solution for the (benefit) transportation problem is  $x_{A1}=10$ ,  $x_{A2}=2$ ,  $x_{B2}=5$ ,  $x_{B3}=10$  and  $x_{ij}=0$  otherwise. The characteristic function of the game  $(N, v^T)$  is the following:

$$\begin{aligned} v^T(N) &= 153; v^T(A123) = 68, v^T(B123) = 85, v^T(AB12) = 110, v^T(AB13) = 120, v^T(AB23) = 100; \\ v^T(A12) &= 68, v^T(A13) = 66, v^T(A23) = 46, v^T(B12) = 70, v^T(B13) = 80, v^T(B23) = 85, v^T(AB1) = 60, \\ v^T(AB2) &= 50, v^T(AB3) = 60; v^T(A1) = 60, v^T(A2) = 40, v^T(A3) = 30, v^T(B1) = 40, v^T(B2) = 50, \\ v^T(B3) &= 60; \text{ otherwise } v^T(S) = 0. \end{aligned}$$

In this case,  $Owenset(N, v^T) = \{(48, 75; 20, 0, 10)\}$ . We know that this allocation is in the core of the game but it seems unfair with retailer 2 since this player contributes significantly to the benefit of the grand coalition, in particular  $v^T(N) - v^T(AB13) = 33$ . As for the core of the game, the segment comprised between the allocations  $(68, 85; 0, 0, 0)$  and  $(7, 17; 53, 33, 43)$  is contained in the core of the game. Therefore,  $Core(N, v^T)$  is larger than  $Owenset(N, v^T)$ . Likewise, if we consider the following two weight systems  $(\Sigma^1, \pi^1) = (\{1, 2, 3\}, \{A, B\}; (1, 1, 1, 1))$  and  $(\Sigma^2, \pi^2) = (\{A, B, 1, 3\}, \{2\}; (1, 1, 53/7, 43/7, 1))$ , then we obtain the following two pairwise solutions  $PS^{(\Sigma^1, \pi^1)}(N, v^T) = \{68, 85; 0, 0, 0\}$  and  $PS^{(\Sigma^2, \pi^2)}(N, v^T) = \{7, 17; 53, 33, 43\}$ . On the other hand, if we simply consider  $\pi = (1, 1; 1, 1, 1)$ , then we obtain the pairwise egalitarian solution  $PS^{(1,1,1,1)}(N, v^T) = \{(34, 42.5; 30, 16.5, 30)\}$  which, in this example, belongs to the core of the game. Finally, if we consider the vector of weights  $\pi = (1, 2; 3, 4, 5)$ , then we obtain the pairwise solution  $PS^{(1,2,3,4,5)}(N, v^T) = \{(16.60, 25.48; 45.00, 23.07, 42.86)\}$  which does not belong to the core of the game.

### 3.2 Three-sided transportation games

A three-sided transportation problem consists of three sets of agents, called producers, wholesalers and retailers, which produce, store and demand goods. Each producer produces an amount of goods, each wholesaler has a capacity of storage and each retailer demands a certain amount of goods. The transport of the goods from the producers to the retailers via a wholesaler is costly (profitable) and, therefore, the main objective is to transport the goods from the producers to the retailers via the wholesalers at minimum cost (at maximum profit). We will call this situation the *distribution problem*. The same reasoning about the interest of cooperation and the benefit approach holds for these problems.

Let  $P$ ,  $W$  and  $R$  be the sets of producers, wholesalers and retailers respectively. We denote by  $p_i$  the production of goods of producer  $i \in P$ ,  $c_j$  the capacity of storage of wholesaler  $j$  and by  $d_k$  the demand of goods of retailer  $k \in R$ . The unitary benefit of transportation from producer  $i$  to retailer  $k$  via wholesaler  $j$  is denoted by  $b_{ijk}$ . The mathematical program that models this problem is the following:

$$\begin{aligned}
 \max \quad & \sum_{i \in P} \sum_{j \in W} \sum_{k \in R} b_{ijk} x_{ijk} \\
 \text{s.t.} \quad & \sum_{j \in W} \sum_{k \in R} x_{ijk} \leq p_i, \quad i \in P \\
 & \sum_{i \in P} \sum_{k \in R} x_{ijk} \leq c_j, \quad j \in W \\
 & \sum_{i \in P} \sum_{j \in W} x_{ijk} \leq d_k, \quad k \in R \\
 & x_{ijk} \geq 0, \quad i \in P, j \in W, k \in R
 \end{aligned} \tag{14}$$

where  $x_{ijk}$  is the number of units transported from producer  $i$  to retailer  $k$  via wholesaler  $j$ . Now, we can define a cooperative game in characteristic function form associated with each distribution problem  $D$ . The set of players  $N = P \cup W \cup R$  and the characteristic function  $v^D$  is defined following the formulas in (8). The game  $(N, v^D)$  is called *distribution game*.

On the one hand, in (Quint, 1991) it is shown that the core of  $m$ -sided assignment games can be empty, therefore if we consider that the goods are indivisible then distribution games can have empty cores. In this sense, there will be many distribution situations in which a core

allocation is not possible. Furthermore, the Owen set could consist of non efficient allocations because the duality gap. However, we can always find reasonable allocations based on the primal optimal solutions of problem (14), defined analogously as pairwise solutions, which we call *triplewise solutions*.

On the other hand, if we consider that the goods are perfectly divisible then distribution games have non-empty cores since the Owen set of these games is always non-empty and it is contained in the core of the game. Of course, in distribution situations with perfectly divisible goods, it is also possible to consider the triplewise solutions as reasonable solutions.

We would like to point out that, in the case of two-sided transportation situation, we have not distinguished between indivisible and perfectly divisible goods because the constraint matrix in problem (10) is totally unimodular and therefore we can relax the indivisibility condition when necessary.

Finally, (Perea et al., 2008) study from a cooperative standpoint a class of distribution problems and prove that the corresponding cooperative games have non-empty core. Likewise, the authors introduce two new solutions which satisfy certain interesting properties related to fairness.

### 3.3 Warehouse sharing games

Now, we consider another situation, also related to transportation problems, in which there are two or more distribution systems, each of them consisting of producers, warehouses and retailers. In principle several producers and retailers could belong to different distribution systems but the warehouses can only belong to one distribution system. In this situation the distribution systems involved in the problem could share their warehouses in order to increase the efficiency of all systems considered as a whole. Therefore, if cooperation is profitable then this should be promoted through a good distribution of the extra profits or saved costs. A similar reasoning about the benefit approach holds for these problems. We will call these optimisation situations *warehouse sharing problems*.

Each distribution system faces the same optimization problem which is modelled as (14). Likewise, if two or more distribution systems collaborate then the corresponding optimisation problem is also modelled as (14). Therefore, we can approach this situation as an operations research game.

Let  $D$  be the set of distribution systems and  $P_i$ ,  $W_i$  and  $R_i$  the sets of producers, warehouses and retailers in distribution system  $i \in D$ . We denote by  $p_{if}$  the production of goods of producer  $f \in P_i$ ,  $c_{ig}$  the capacity of storage of warehouse  $g \in W_i$  and by  $d_{ih}$  the demand of goods of retailer  $h \in R_i$ . The unitary benefit of transportation from producer  $f \in \bigcup_{i \in D} P_i$  to retailer  $h \in \bigcup_{i \in D} R_i$  via warehouse  $g \in \bigcup_{i \in D} W_i$  is denoted by  $b_{fgh}$ . If one producer (resp. retailer) belongs to more than one distribution system then, when these distribution systems collaborate, the production (resp. demand) to take into account for that producer (resp. retailer) is the sum of its productions (resp. demands). As it is not difficult to see, the mathematical formulation of this problem is as (14).

Next, we can define a cooperative game in characteristic function form associated with each warehouse sharing problem  $WS$ . In this case, the set of players  $N = D$  and the characteristic function  $v^{WS}$  is defined following the formulas in (8). The game  $(N, v^{WS})$  is called *warehouse sharing game*.

In this kind of situation we can observe two levels of cooperation. On the one hand, we find the cooperation among producers, warehouses and retailers inside of a distribution system. And, on the other hand, we have the cooperation among the different distribution systems. It is obvious that if we are only interested in the warehouse sharing game then similar comments as in Sections 3.1 and 3.2 regarding the allocation of the extra benefits among the agents involved can be done. However, if we are interested in the two levels simultaneously considering the problem as a whole system then, perhaps, we may be dealing with a game with a priori unions or restricted cooperation and, consequently, we should take into account this fact in order to analyse this situation.

Finally, this situation can resemble the cooperation among supply chains with deterministic productions/demands and without penalties and, therefore, it could be considered within of the literature of supply chain games. However, we have considered its analysis more appropriate as an operations research game because the mathematical model describing this problem is close related to a three-sided transportation situation as we have shown. On the other hand, several papers, in which different levels of cooperation (horizontal, vertical or lateral) are analysed for transportation or supply chain situations, are (Crujssen et al., 2007), (Mason et al., 2007) and (Simatupang & Sridharan, 2002).

#### **4. Supply chain games**

For researchers in Operations Research and Economics, supply chains represent one of the key issues which can be relied on. This section brings together a series of works, which present different paradigms and results related to cooperative game theory as applied to supply chain management. This comprises review oriented papers that look at the kind of methodologies that have been applied, in addition to theoretical papers discussing new developments and results. As a direct consequence of this, we hope that this section will serve as a source for current and future researchers in this field.

Moreover, another aim of this part is to show the applicability of cooperative game theory as a tool with which to analyse supply chains since a main feature of any supply chain is cooperation. In particular, the central contribution of cooperative game theory is related to determine a suitable allocation rule among the agents of that supply chain. However, we would like to point out that the use of cooperative game theory to analyse problems in supply chain management is a very recent development.

##### **4.1 Definition of a supply chain**

There are numerous definitions for the term "supply chain". For example, (Christopher, 1998) defined this notion as "... network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer". Whereas (Ganeshan et al., 1999) define a supply chain as "a system of suppliers, manufacturers, distributors, retailers and customers where materials flow downstream from suppliers to customers and information flows in both directions". On the other hand, supply chain management is defined as a set of management processes (Leng & Parlar, 2005). However, all definitions in the literature share the idea that supply chains are based on cooperation in order to obtain a higher benefit. In fact, (Thun, 2005) claims that, in the future, competition will take place between supply chains instead of between individual firms. In order to yield

the benefits related to cooperation, contracts for vertical cooperation must be established within supply chains.

Nevertheless, the main drawbacks for the right supply chain management are two. First, trust can be seen as the most critical factor of cooperation between firms (Poirer, 1999). In this way, modelling supply chains via cooperative games can be important to analyse the impact of rationality on the final allocation (Thun, 2005). Secondly, there is a phenomenon commonly referred to as “the bullwhip effect”, which was first observed at P&G concerning disposable diapers (Lee et al., 1997). Sharing information across the supply chain is a way to mitigate its negative effects (Thun, 2005).

#### 4.2 Examples of supply chain games

In this section we show two examples of situations related to supply chain management. The first example is based on (Müller et al., 2002), while the second one is based on (Granot & Sosic, 2003).

**Example 1.** We consider the usual newsvendor game where each agent (store) faces a stochastic demand (of newspapers, for example). These demands are actually correlated, although this fact has usually been ignored in the literature seeking simplicity. We will take into account this feature of the game. So, any coalition of agents that faces a demand  $x$  and orders a quantity  $y$  of newspapers incurs a cost as follows,

$$\phi(y, x) = \begin{cases} h(y - x), & \text{if } y \geq x \\ \pi(x - y), & \text{if } y < x \end{cases} \quad (15)$$

where  $h$  is the holding cost per unit of stocking more newspapers than are actually demanded, and  $\pi$  is the opportunity cost related to not ordering enough newspapers. Following with the description of the game, each agent  $i$  experiences a random demand  $X_i$ . For coalition  $S \subset N$ , we define the total demand as  $X_S = \sum_{i \in S} X_i$ . For technical purposes, we

focus on random demands such that  $E[\phi(y, X)] < \infty$ . In this way, the optimal quantity ordered by  $S$  is  $y_S^* = \arg \min_y E[\phi(y, X_S)]$  and coincides with the  $\pi/(h + \pi)$  quantile of the distribution of the random variable  $X_S$ . Consequently, the value (cost) of the characteristic function of coalition  $S$  in this kind of game is defined as  $C(S) = E[\phi(y_S^*, X_S)]$ . Finally, let  $N$  be the finite set of agents. In this way, we are able to define a cooperative game as  $(N, C)$ .

**Example 2.** In this example we briefly show a three-stage game of a supply chain consisting of  $n$  retailers, each of whom experiences a random demand for an identical product. Next we explain the different steps of the game. Before the demand is realised, each player orders her initial inventory in an independent way (first stage). After the demand is actually realised, each player decides how much of their residual stock they wish to share with the other retailers (second stage). In the final stage, a total profit should be allocated among the players due to the fact that residual stocks are transhipped to meet the joint demand. In this way, in the third (cooperative) stage, residual inventories are transhipped to meet residual demands, and the additional profit has to be allocated among the retailers. Obviously, this example excludes the possibility of storing at one or several shared warehouses.

### 4.3 Review of the literature on supply chain games

Many articles on supply chain management point towards the relevance of cooperation among the supply chain members in order to increase the supply chain benefits and the overall performance. However, only a few researchers so far have deployed cooperative game theory to analyse the stability and rationality of collaboration within a supply chain. Authors such as (Cachon & Netessine, 2004) have reviewed the literature describing supply chain and game theory concluding that “papers employing cooperative game theory have been scarce, but are becoming more popular”. Something similar has been pointed out in other reviews such as (Leng & Parlar, 2005) and (Nagarajan & Sobic, 2008). This section is partially based on these good reviews. Nevertheless, we have added very recent publications on this issue which were not mentioned in those three reviews. On the other hand, for a specific review of the literature on inventory centralization we refer to (Meca & Timmer, 2008).

In 1961 (Chacko, 1961) analysed the impact of coalition formation between a multi-plant multi-product manufacturing company, two suppliers and several customers. Unfortunately, this paper did not become the starting point for the use of cooperative game theory in supply chain. Twenty years later, one can find a paper mixing supply chain management and cooperative game theory. (Jeuland & Shugan, 1983) explored the problem of coordination of the members of a channel, which includes as a particular case the manufacturer-retailer-consumer channel. They also proposed the form of the quantity discount schedule that results in optimum channel profits. (Kim & Hwang, 1989) studied how the supplier can formulate the terms of a quantity-discount pricing schedule, under the assumption that the supplier behaves in an optimal way. In particular, they show the formula for price and order size that maximises the sum of the profits of both agents and the corresponding allocation between the parties.

(Gerchak & Gupta, 1991) analysed the effectiveness of four popular schemes of cost allocation in the context of a continuous review order quantity reorder point  $(Q, r)$  inventory system with complete back ordering. They also proposed a proportional method that has the notable feature that any customer's post-centralization share of overheads does not exceed its costs without consolidation. Inspired by this paper, (Robinson, 1993) showed that the best allocation rule proposed in (Gerchak & Gupta, 1991) does not necessarily belong to the core. Furthermore, he also showed the formulation of the Shapley value for this game and proved that this allocation rule does actually belong to the core.

(Wang & Parlar, 1994) proposed a single-stage game to model a particular inventory problem where three retailers try to determine their optimal order amount. They assume stochastic demands and substitutable products. In this context, they determine the conditions that assure that the core of this game is non-empty.

So far the papers reviewed focus on horizontal cooperation in a supply chain. Nevertheless, there are papers devoted to vertical cooperation. One example is the paper by (Li & Huang, 1995). They explored the simple (monopolistic) buyer-seller channel from a cooperative approach. The authors showed the common incentives and the individual disincentives for cooperation. A rule, based on quantity discount, is also proposed to implement a profit sharing mechanism for achieving equal division of additional cooperative system profits.

In (Hartman & Dror, 1996) the cost allocation problem for the centralized and continuous-review inventory system is studied. They proposed three necessary criteria (stability, justifiability and polynomial computability) for appropriating selection of an allocation rule.

They showed that common allocation schemes may not meet the three criteria and introduced a method that meets them all. Following this line, (Hartman et al., 2000) considered a set of  $n$  stores with centralized ordering and inventory with holding and penalty costs. They showed the (restrictive) condition under such a cooperative game has a non-empty core and conjectured that the core is non-empty at least for independent demands. (Hartman & Dror, 2003) proved the non-emptiness of the core for a single period inventory game with  $n$  retailers experiences normally distributed, correlated individual demands. On the other hand, (Müller et al., 2002) proved a stronger result than that conjectured by (Hartman et al., 2000). In particular, they showed that the core of this type of games is always non-empty regarding the joint distribution of the stochastic demands. (Slikker et al., 2005) studied a more complex situation, called the general newsvendor game, where the agents could use transshipments after demand is satisfied. Their main result states that the general newsvendor game has a non-empty core.

(Anupindi et al., 2001) analysed a supply chain problem with  $n$  independent retailers of an identical item for consumption. Each agent experiences a random demand and must order their inventory before the demand is realised. After realising such a demand, some retailers might meet their residual demand by means of the other retailers' residual supplies. This game is very similar to example number 2 above. Nevertheless, it is played as a decentralised two-stage distribution model, whereas example 2 consists of three stages. In addition, (Anupindi et al., 2001) assumed that all retailers will share all their residual supply/demand in the second stage. Regarding the allocation schemes, these authors suggested an allocation rule based on a dual solution for the transshipment problem. This solution is always in the core of the game and, hence, it encourages the retailers not to form coalitions. Later, (Granot & Sosic, 2003) extended the two-stage model of (Anupindi et al., 2001) allowing each retailer to decide how much of their residual supply/demand they would like to share with others in a third and final stage. They found that allocations based on dual solutions will not induce the retailers to share their total residuals with others. Furthermore, they proved that the Shapley value is a value-preserving allocation scheme, i.e., it induces all the retailers to share their residual supply/demand in quantities that do not result in a decrease in the total additional profit.

We now turn to vertical cooperation in supply chain problems and consider the paper of (Raghunathan, 2003). This author studied a situation where a manufacturer and  $n$  retailers share demand information. The author used the Shapley value to analyse the expected manufacturer and retailer shares of the surplus generated from the cooperative game. Mainly, (Raghunathan, 2003) showed that higher demand correlation increases the manufacturer's allocation and has the opposite result on the retailers.

Under horizontal cooperation, (Meca et al., 2004) studied a simple inventory model with  $n$  retailers who experience deterministic demand. The firms can cooperate to reduce their ordering costs. This approach is called the basic inventory model because it forms the basis for a wide variety of inventory models. Also, the authors developed a proportional rule to allocate joint ordering cost among the retailers. They showed that this rule leads to an allocation in the core. For a more general study of holding games see (Meca, 2007).

(Hartman & Dror, 2005) studied the problem faced by the management of independent stores, with a similar product, of cost management for a centralised operation of their inventory. They modelled the centralised cost as a metric space obtained from the Cholesky factorisation of the corresponding covariance matrix. They considered two cooperative

games, one based on optimal expected costs and another based on demand realisations. For the first game, they showed that when holding and penalty shortage costs are identical and normally distributed demands, the corresponding game has a non-empty core. Unfortunately for the second game, they showed that even in the case of identical holding and penalty costs the game might have an empty core.

(Klijn & Slikker, 2005) analysed a location-inventory model with  $m$  customers and  $n$  distribution centres. Under this context, they proved the emptiness of the corresponding cooperative game when demand processes are identically and independently distributed.

(Reinhardt & Dada, 2005) considered a problem with  $n$  firms who collaborate by pooling their critical resources in order to make their cost structure more efficient. They proposed to use the Shapley value as the allocation scheme among the players. For coalition symmetric games, i.e., situations where the pooled savings depend on the sum of each player's demand, they introduced a pseudo-polynomial algorithm for its computation.

In a vertical cooperation framework, (Leng & Parlar, 2005) analysed an information-sharing cooperative game involving a supplier, a manufacturer and a retailer. They derived the necessary conditions for stability of each coalition. They also studied the implications of using the Shapley value and the nucleolus as allocation schemes for this type of games.

More recently, (Dror & Hartman, 2007) analysed cost allocation in a multiple product inventory system following an economic order quantity policy to order, where part of the ordering cost is shared and part is specific to each item. They showed that if the part of the ordering cost common to all items is not too small, then the core of the game is non-empty.

(Montrucchio & Scarsini, 2007) considered a newsvendor game with stochastic demand of a single item. They proved that the game is balanced in great generality considering a possibly infinite number of retailers. Under several conditions, they also showed that with a continuum of retailers the core becomes a singleton.

Under vertical cooperation, (Guardiola et al., 2007) analysed a supply chain under decentralised control with a single supplier and  $n$  retailers. They proved that the cooperation in this game is stable and proposed a specific allocation rule that is always in the core. This last point is important since the well-known Shapley value does not always belong to the core for this type of games.

(Guardiola et al., 2009) introduced a new class of production-inventory games. Cooperation among agents is given by sharing production processes and warehouses facilities. In this context, the authors proved that the corresponding cooperative game is totally balanced and the set of the Owen-allocations is a point (called the Owen point). Also, the authors showed the relationship between the Owen point, the Shapley value and the nucleolus.

(Özen et al., 2008) conducted a game-theoretical analysis of a supply chain with warehouses, in which retailers have the chance of reallocating their product orders after the demand has been met. In this context, the authors considered a cooperative game between the retailers. They were able to prove that this game has a non-empty core.

(Chen & Zhang, 2009) demonstrated the power of stochastic programming duality approach in studying stochastic inventory games. In fact, their approach is readily applicable to more general models. In this context, as a main result, they showed that stochastic programming provides a way to compute a solution in the core of this kind of games.

Finally, (Özen et al., 2010) considered a simple newsvendor game and investigated the convexity of this type of situations. Whereas it is known that the general newsvendor game is not convex, they focused on the particular family of newsvendor games with independent



symmetric unimodal demand distributions. It allowed them to identify several interesting subclasses containing convex games only.

#### 4.4 Further research in supply chain management

We devote this section to suggesting several avenues for further follow-up research in cooperative supply chain games. To this end, we show two interesting contexts related to current and real supply chains. The first is based on (Plambeck & Taylor, 2005) and shows the benefits from collaborating between a pharmaceutical company and a manufacturer. The second context is inspired by the actual Spanish electricity market. We propose to analyse the cooperation between electricity consumers, retailers and the network operator by means of cooperative game theory. In a certain sense, such a framework generalises the approach introduced in (Pettersen et al., 2005) for a single consumer, a single retailer and the network operator in the Nordic electricity market. It is worth mentioning that both contexts are not related to holding costs and inventory problems, a feature that is not usual in the supply chain literature, as we have shown previously.

##### 4.4.1 Contracting manufacturers in the pharmaceutical industry

As pointed out in (Plambeck & Taylor, 2005), firms in the pharmaceutical industry are characterised by long developments cycles and intensive time-to-market pressure. In this industry, any firm that produces its own drug must make a significant capital investment in a plant before the product has completed regulatory trials. Unfortunately, if the drug finally fails, then the plant belonging to the pharmaceutical company (PC) will have little value (Tully, 1994). This drawback is usual in industries where production capacity is low in contrast to their investment power. In this case, contract manufacturing offers the opportunity to outsource production to contract manufacturers (CMs). They are able to pool the total demand from many different pharmaceutical companies and, consequently, achieve high capacity utilization.

Following (Plambeck & Taylor, 2005), we consider two symmetric PCs,  $j=1, 2$ , which are developing a new drug. The price per unit when  $q_j$  units are sold is  $M_j - q_j$ . With probability  $e$ , the product is successful and  $M_j = H_j$  where  $H_j$  represents the potential market size. Otherwise,  $M_j = L$  with  $L < H_j$ . On the other hand, each PC should invest in production capacity  $c$  at a cost of  $k > 0$  per unit before the demand is known. Furthermore, the marginal cost of production is negligible.

Investments by the PCs in innovation (product development) may influence demand through in two ways. On the one hand, increasing the potential market size,  $H_j$ . On the other hand, the probability that a drug passes clinical trials influences positively the final success probability. We here consider the first case, i.e., when investment in innovation influences  $H_j$ . So, let  $f(H_j)$  be the total cost function of innovation of firm  $j$ . It is also assumed that this function is increasing at the market size, twice differentiable and convex. Each PC selects a market size  $H_j$  that maximizes its total expected profit,  $V_j$ .

$$V_j = \max_{H_j} \left\{ \max_{c \geq 0} \left\{ e(H_j - c)c + (1 - e) \max_{q_j \in [0, c]} \left\{ (L - q_j)q_j - kc \right\} \right\} - f(H_j) \right\}. \quad (16)$$

Consider now that the two PCs pool their production capacity in this game ( $c + c = 2c$ ). In other words, we assume that  $\{PC_1, PC_2\}$  is a coalition. In this way, the maximum expected profit that they can achieve is

$$V_{\{1,2\}} = \max_{H_1, H_2} \left\{ \max_{c \geq 0} \{R(c, H_1, H_2) - 2kc\} - f(H_1) - f(H_2) \right\}, \quad (17)$$

where

$$R(c, H_1, H_2) = e^2 \max_{\substack{q_1, q_2 \geq 0 \\ q_1 + q_2 \leq 2c}} \{ (H_1 - q_1)q_1 + (H_2 - q_2)q_2 \} + \\ e(1-e) \sum_{j=1,2} \max_{\substack{q_H, q_L \geq 0 \\ q_H + q_L \leq 2c}} \{ (H_j - q_H)q_H + (L - q_L)q_L \} + \\ (1-e)^2 2 \max_{q \in [0, c]} \{ (L - q)q \}. \quad (18)$$

We now turn to the situation where an independent CM (player number 3) possesses the capability for producing. We consider that the CM invests in production capacity at a cost of  $k_{CM}$  per unit, with  $k_{CM} < k$ . Therefore, we are considering a situation slightly different of that in (Plambeck & Taylor, 2005).

It is obvious that the CM alone achieves profit zero. This type of firm needs to collaborate with at least one PC to get a strictly positive profit through the production of the final product. Then, the joint profit for the coalition  $\{j, 3\}$ ,  $j=1, 2$ , is equivalent to  $V_j$  with  $k_{CM}$  instead of  $k$ . In the same manner, the profit of the grand coalition would be equivalent to  $V_{\{1,2\}}$  with  $k_{CM}$  instead of  $k$ . Cooperative game theory is the natural way to allocate the value of the grand coalition among all firms. In particular, it could be interesting to analyse stability of cooperation between the pharmaceutical companies and the manufacturer and to look for reasonable and fair distribution of the extra benefits among them.

#### 4.4.2 Supply chain without storage: electricity games

Following the description of the Spanish Electricity Market we propose several games which could be interesting to study. These games have the special feature that the electricity cannot be stored and, therefore, in this context there is not holding or inventory costs. This aspect is not usual in the supply chain literature.

In 1998, the Spanish government liberalised the market for generating electricity and introduced a spot market for electricity. The basic design of this electricity spot market is similar to the previously deregulated UK market and even closer to the Californian electricity market that was deregulated at about the same time. A liberalised electricity market was not new to Spain, as during the 1990s there had been a previous liberalisation of other sectors, such as the media, telephony, oil and gas. In spite of the fact that deregularisation was a slow process which was not completed until 2009, it was not a process that provided the electricity market with a large number of companies selling energy to small consumers of power. The present situation in Spain continues to be one with few companies on the market which stimulate competition and thereby bring about the expected reduction in prices. The main characteristics of the Spanish electricity sector are the existence of the wholesale Spanish generation market (Spanish pool), and the fact that all consumers are considered to have qualified since 2003. This means that they can choose the electricity company that supplies them with electricity and therefore participate in the pool in an active manner. The electricity production market in Spain is organised around a series of auctions and technical procedures for operating the system: Daily Market, Intradaily

Market, Bilateral Contracts, International Contracts, Technical Constraints, Technical Management, etc. (see, for example, (Sancho et al., 2008)). Since 2006, bilateral contracts and the forward market have become a larger part of the market. On the other hand, generation facilities in Spain operate either under the Spanish ordinary regime or the Spanish special regime. The electricity system must acquire all electricity offered by special regime generators, which consist of small or renewable energy facilities, at tariffs fixed by Royal Decree or Order that vary depending on the type of generation and are generally higher than Spanish market prices. Ordinary regime generators provide electricity at market prices to the Spanish pool and under bilateral contracts to qualified consumers and other suppliers at agreed prices. Suppliers, including last resort suppliers, and consumers can buy electricity in this pool. Foreign companies may also buy and sell in the Spanish pool. The market operator and agency responsible for the market's economic management and bidding process is the Electrical Market Operator (OMEL - [www.omel.es](http://www.omel.es)). Market participants are undertakings that are authorised to act directly in the electric power market as buyers and sellers of electricity. The following can be market participants:

- Electric power distributors who come to the market to purchase the electricity needed to supply consumers at regulated tariffs or to distributors who are supplied.
- Resellers: They go into the market to purchase power to sell to qualified consumers.
- Qualified consumers: They can purchase power directly in the organised market, through a reseller, by signing a physical bilateral contract with a producer or by continuing temporarily as a regulated tariff consumer.

Transmission companies and regulated distributors must provide network access to all consumers that have chosen to be supplied on the free market. However, these consumers must pay an access tariff to the distribution companies if such access is provided. The electricity transport grid comprises transmission lines, stations, transformers and other electrical equipment with a voltage superior to 220 KV, as well as other facilities, regardless of their voltage, that provide transport or international and extra-peninsular interconnections. Red Eléctrica de España (Spanish Electrical System Operator), REE - [www.ree.es](http://www.ree.es), manages most of the transmission network in Spain. It is responsible for the technical management of the Spanish electricity system with regards to developing the high voltage network, in order to guarantee electricity supply and proper coordination between the supply and transmission system, as well as the management of international electricity flows. The system's operator carries out its duties in coordination with the market operator. Liberalised suppliers are free to set a price for their consumers. The main direct activity costs of these entities are the wholesale market price and the regulated access tariffs to be paid to the distribution companies. Electricity generators and liberalised suppliers or qualified consumers may also engage in bilateral contracts without participating in the wholesale market. As from 2009, last resort suppliers, appointed by the Spanish government, supply electricity at a regulated tariff set by the Spanish government to the last resort consumers (low-voltage electricity consumers whose contracted power is less than or equal to 10 KW). Since then, distributors cannot supply electricity to consumers.

All generation facilities that are not governed by the Spanish special regime are governed by the Spanish ordinary regime. Under said ordinary regime, there are four methods of contracting for the sale of electricity and determining a price for the electricity:

- Wholesale energy market or pool. This pool was created on January 1, 1998 and includes a variety of transactions that result from the participation of market agents

- (including generators, distributors, suppliers and direct consumers) in the daily and intraday market sessions.
- Bilateral contracts. Bilateral contracts are private contracts between market agents, whose terms and conditions are freely negotiated and agreed.
  - Auctions for purchase options or primary emissions of energy. Principal market participants are required by law to offer purchase options for a pre-established amount of their power. Some of the remaining market participants are entitled to purchase such options during a certain specified period.
  - Energy Auctions for Last Resort Demand. Last resort suppliers in the Iberian Peninsula can acquire electricity in the spot or forward markets to meet last resort demand. However, beginning in 2007, these last resort suppliers were permitted to begin holding energy auctions to purchase electricity at lower prices. Since 2003, all consumers have become qualified consumers. All of them may now choose to acquire electricity under any form of free trading through contracts with suppliers, by going directly to the organised market or through bilateral contracts with producers.

With the coming into force of the Last Resort Supply in 2009, the integral tariff system has been replaced by a last resort tariff system. Last resort tariffs are set on an additive basis and can only be applied to low-voltage electricity consumers whose contracted power is less than or equal to 10 KW. Last resort consumers can choose either to be supplied at last resort tariffs or to be supplied in the liberalised market.

Within the regulatory framework, it is important to point out that there is very low, almost insignificant, participation in the Spanish electricity market by small and medium consumers. To this end, over the last few years, different independent system operators (ISOs) in Europe, Oceania and North America are continuing the development of load response programmes (LRPs) with the objective of changing electricity demand of large power users. Nevertheless, some medium commercial or industrial users may submit offers and bids in new energy markets thanks to lighter requirements for demand reduction with levels of about 100kW (New York ISO or New England ISO). In addition, some ISOs encourage the possibility of demand aggregation through commercial entities (see pilot programmes developed by NYISO since 2002 for small load aggregators (ISO New England Market - [www.iso-ne.com](http://www.iso-ne.com)) to reach the minimum level for the participation of users. As with these international markets, in the medium term, commercial and aggregating companies will have to offer users in Spain a selling price for power that fits in with the consumption profile of a specific segment of customers (Verdu et al., 2006). They must also offer customers various participation schemes in the demand which will allow the electricity companies to group together sufficient levels of power to be able to buy energy on the electricity market. At the same time, customers signing up to the schemes will receive special offers to reduce or modify their consumptions levels (Valero et al., 2007).

After reading the description of the Spanish Electricity Market it is possible to think that different games could be analysed. For example, in the literature there are many papers analysing from a game theoretical standpoint the electricity auction-market (see, for example, (Aparicio et al., 2008) and (Sancho et al., 2008) and their lists of references). Another interesting problem is the game played by electricity consumers, retailers and the network operator. In (Pettersen et al, 2005) this game for only one electricity consumer, one retailer and one network operator is studied from a non-cooperative point of view. A generalisation of this approach could be to consider a higher number of agents involved in

the game. Alternatively, this game could be studied from a cooperative point of view by restricting the possibilities of cooperation in order to respect some level of competition in the market.

Taking into account the possibility of bilateral agreements in the electricity market, the horizontal cooperation among users or consumers could be an interesting problem to be studied from a game theoretical point of view since, at first sight, collaboration among the consumers could be profitable for them because, perhaps, all together could obtain better electricity prices. In this context, we could consider two sides of the electricity market. One side of the market would consist of the suppliers of electricity who should compete for selling electricity. The other side of the market would consist of the consumers who could collaborate in order to get a better position in the market. The analysis of this situation could provide insights on the level of competition among the suppliers and the interests of cooperation among the consumers.

The last game we would like to mention in this part is related to vertical cooperation. At first glance the functioning of the electricity market with respect to small or renewable energy facilities seems appropriate because the market is promoting the use of green energy. However, this could provoke inefficiencies in the system such that a loss of productivity in the firms because of a higher electricity cost. Therefore all agents involved in the electricity market should collaborate in some sense. Of course, this cooperation should not imply a loss of competition in the market but a re-structuring of some aspects of it, for example, the determination of different quotes of electricity production depending on the energy source. Likewise, in the analysis of this problem, the CO<sub>2</sub> market implications or the production of obnoxious residues might also be taken into account. In this situation, perhaps, a cooperative game theoretical approach could be used in order to obtain some insight about the electricity market.

## 5. Other logistics games

There are a considerable number of papers concerned with other situations related to logistics problems. In this section we show some of these works as an example of the magnitude and relevance of cooperative game theory in this question. In particular, we focus on routing, packing and location games. For each category we will present some approaches trying to illustrate their relationship with logistics. For this reason, we will pay special attention to the modelling stage. In other words, we will try to explain how to go from the logistics problem to cooperative game theory. Also, we will show the main results of each contribution. For a specific revision of the literature on connection and routing problems and cooperative game theory we refer to (Borm et al., 2001).

We start with a couple of problems related to routing (see (Borm et al., 2001), (Hamers et al., 1999) and (Potters et al., 1992)). First, we will study the classical Chinese postman game. Second, we will discuss the well-known travelling salesman game. Both problems are related to the logistics problem of how to design efficient routes to deliver the commodities from the supply nodes to the demand nodes.

In the classical *Chinese postman situation*, a postman must deliver mail to each street of a city. Obviously, she has to start and finish at the post office. Moreover, each street has an associated cost, related to the time that the postman expends in each visit. The aim in this problem is to select the optimal route. To describe mathematically this situation we need a 4-tuple  $\langle N, G, v_0, t \rangle$ , where  $N$  is the set of players (streets),  $G = (V, E)$  is a connected undirected

graph with vertex set  $V$  and edge set  $E$ ,  $v_0 \in V$  is the post office and  $t$  is a nonnegative cost function. We denote a route for coalition  $S \subset N$  as  $(v_0, e_1, \dots, e_k, v_0)$ , which starts and finishes at the post office and visits each player in  $S$  at least once. Finally,  $D(S)$  represents the set of all routes for coalition  $S$ .

The *Chinese postman game*  $(N, c)$  associated with the 4-tuple  $\langle N, G, v_0, t \rangle$  is defined from the following cost function for every coalition  $S \subset N$ .

$$c(S) = \min_{(v_0, e_1, \dots, e_k, v_0) \in D(S)} \left\{ \sum_{j=1}^k t(e_j) - \sum_{i \in S} t(i) \right\}. \quad (19)$$

One result we would like to highlight is that this type of games need not be balanced. For this reason, the Chinese postman game has been studied in the literature under several additional constraints on the underlying graph: efficiency, bridge cluster symmetry, condensation property and so on.

Regarding another routing situation, the *travelling salesman problem* is similar to the Chinese postman problem but in this case there are a set of cities (vertices or nodes) which have to be visited by the salesman and each link connecting two cities has a cost (distance, time, etc.). The objective is to determine a route or tour that visits each city exactly once at minimal cost. The travelling salesman problem can be described formally by means of a triple  $\langle N, 0, t \rangle$ , where  $N$  is the set of player as usual, 0 represents the home location and  $t$  is a nonnegative cost function. The costs match the edges linking the vertices in  $N \cup \{0\}$ . In this case, the characteristic function of the cooperative game, which could be generated from the travelling salesman problem, coincides with the minimal cost of a Hamiltonian circuit in the graph associated with each coalition  $S$ . This type of game needs not be balanced, i.e., the core could be empty. Nevertheless, (Potters et al., 1992) showed that the *travelling salesman game* with three players have a non-empty core. Other authors have proved that games with four and five players are balanced as well (see (Borm et al., 2001)).

We now turn to a different class of games: *packing games*. Imagine a set of manufacturers, called  $A$ , and a set of transport companies, called  $B$ . Each firm  $i \in A$  has an item of size  $a_i$ , while each individual in  $B$  possesses a truck of capacity  $b_j$ . The items yield a profit proportional to their size. Nevertheless, it is necessary for each item to be brought to a certain market by means of a truck. Moreover, we assume that each truck can make only one trip to the market. We can define a packing as an assignment of some items in  $A$  to the trucks in  $B$  such that the total size does not exceed the total truck capacity. The value of a packing coincides with the sum of the sizes of all packed items. In this way, a bin packing problem has as a goal to determine a packing of maximal value. Cooperative game theory tries to share the total profit among the individuals of sets  $A$  and  $B$  in a reasonable way. (Faigle & Kern, 1993) introduced these games in the literature. They studied the emptiness of the core, showing that (bin) packing games may be not balanced. Due to this fact, (Faigle & Kern, 1993) used a generalisation of the core notion, called the  $\varepsilon$ -core. The  $\varepsilon$ -core of a game  $(N, v)$  is defined as

$$\varepsilon\text{-core}(v) = \left\{ x \in R^n : x(N) = v(N), x(S) \geq (1 - \varepsilon)v(S), \forall S \subseteq N \right\}. \quad (20)$$

Using this concept, (Faigle & Kern, 1993) proved that if  $v(N) \geq 0$ , then the  $\varepsilon$ -core is non-empty for a value of  $\varepsilon$  sufficiently large. Following (Faigle & Kern, 1993), (Kuipers, 1998)

showed what is the value of the minimal  $\varepsilon$  such that the  $\varepsilon$ -core is nonempty. Also, this author studied, for a specific class of packing games, the minimal  $\varepsilon$  such that all games in this special class have a nonempty  $\varepsilon$ -core. Also, for computational purposes, it is worth noting that general bin packing situations are NP-complete problems. Nevertheless, the constraint that all trucks have capacity 1 and that all items are strictly larger than  $1/3$  makes the problem easier to solve.

For a more recent study of packing games and their applications see (Sanchez-Soriano et al., 2002). There the authors analysed the transport system for university students in the province of *Alacant* (Spain). The question is how to connect different villages and towns in *Alacant* efficiently with the different university campuses. The authors proposed a possible approach to model this situation. They also considered a particular cost sharing rule based on the egalitarian solution.

In a realistic logistics problem, as the previous one, we could combine both the routing problem and the packing problem because in some way they are closely related. In these situations, we would be interested in determining the number of trucks or containers, taking into account their capacities, and their routes to deliver the different possible commodities from the supply nodes to the demand nodes at minimal cost. Of course, a previous logistics problem, which could be considered, is the location of warehouses or factories in order to improve the efficiency of a posterior delivery chain which would be related to the combination of the routing and packing problems.

So, next, we briefly discuss *location games*. (Puerto et al., 2001) introduced a family of cooperative games arising from continuous single facility location problems. In such a situation, there are  $n$  users of a certain facility (for example, a hospital), placed in  $n$  different points (towns) in  $R^m$ ,  $m \geq 1$ . In this structure, the costs depend on the distances from the users to the facility. We seek a location in  $R^m$  for the facility that minimises the total transportation cost. (Puerto et al., 2001) showed two sufficient conditions so that their location game has a non-empty core. Also, they studied under which conditions the proportional egalitarian solution provides core allocations for Weber and minimax (continuous) location games. More recently, (Goemans & Skutella, 2004) deeply analysed non-continuous location games. In such a problem, there is a set of  $F$  possible locations for the facility/facilities and we have to decide which facility/facilities to build. In addition, each user must be connected to an open facility. Both opening facilities and connecting users have a fix cost. As above, the goal is to minimise the total cost of the system. In this context, (Goemans & Skutella, 2004) established strong links between fair cost allocations and linear programming relaxation. In particular, they proved that a fair cost allocation exists if and only if there is no integrality gap for a corresponding linear programming relaxation. What is much more interesting is that they also showed that it is in general NP-complete to decide whether a fair allocation scheme exists and whether a given cost rule is fair.

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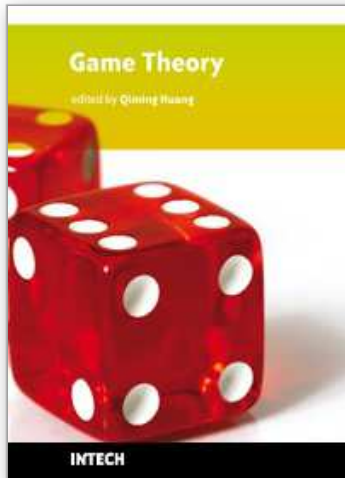
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## **Game Theory**

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Game theory provides a powerful mathematical framework that can accommodate the preferences and requirements of various stakeholders in a given process as regards the outcome of the process. The chapters' contents in this book will give an impetus to the application of game theory to the modeling and analysis of modern communication, biology engineering, transportation, etc...

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